
Recent progress in convex and discrete geometry

Progrès récents en géométrie convexe et discrète

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Grünbaum's inequality for probability measures

Given a convex body K in $\mathbb{R}^n$, a natural question is: if one partitions the body into two pieces along its barycenter, how small can each piece be? By “partition along its barycenter”, we mean intersecting K with a half-space whose boundary is a hyperplane containing said barycenter. Grünbaum showed that the volume of each piece is at least $\left(\frac{n}{n+1}\right)^n$ times the total volume of K . Furthermore, this constant is sharp: there is equality if and only if K is a cone, which means there exists a $(n-1)$ -dimensional convex body L and a vector b , such that K has face L and vertex b (i.e. K is the convex hull of b and L).

In this work, which is joint with M. Fradelizi, J. Liu, F. Marin Sola, and S. Tang, we are interested in generalizing Grünbaum's inequality to other measures. Our main results are a sharp inequality for the Gaussian measure and a sharp inequality for s -concave probability measures. The characterization of the equality case is of particular interest. Along the way, we discover new facts about the equality case of the Borell-Brascamp-Lieb inequality.

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C^1 -maximizer of p -mean torsion rigidity on convex bodies

Given a bounded domain $B \subset \mathbb{R}^{n \geq 2}$ with its boundary ∂B , a solution u_B of the torsion problem

$$\begin{cases} \Delta u_B = -1 & \text{in } B; \\ u_B = 0 & \text{on } \partial B, \end{cases}$$

is called a stress function of B . Via the torsion rigidity

$$\int_B |\nabla u_B(x)|^2 dx,$$

this talk is about to show that the maximization problem for $[1, \infty) \ni p$ -mean torsion rigidity

$$(\star) \quad \sup_{\text{all convex bodies } B \subset \mathbb{R}^n} \int_B \left(\frac{|\nabla u_B(x)|^2}{|B|^{\frac{2}{n}}}} \right)^p \frac{dx}{|B|},$$

is achievable and the boundary $\partial B_\bullet$ of any maximizer $B_\bullet$ of $(\star)$ is C^1 -smooth, thereby finding that if $|\nabla u_{B_\bullet}|$ is constant on $\partial B_\bullet$, then $B_\bullet$ is a Euclidean ball.

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