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C^1-maximizer of p-mean torsion rigidity on convex bodies

Given a bounded domain $B \subset \mathbb{R}^{n\geq 2}$ with its boundary ∂B, a solution u_B of the torsion problem

$$\begin{cases} \Delta u_B = -1 & \text{in } \ B; \\ u_B = 0 & \text{on } \ \partial B, \end{cases}$$

is called a stress function of B. Via the torsion rigidity

$$\int_B |\nabla u_B(x)|^2 \, dx,$$

this talk is about to show that the maximization problem for $[1, \infty) \ni p$-mean torsion rigidity

$$(\star) \quad \sup_{\text{all convex bodies } B \subset \mathbb{R}^n} \int_B \left(\frac{|\nabla u_B(x)|^2}{|B|^{\frac{2}{n}}} \right)^p \frac{dx}{|B|},$$

is achievable and the boundary $\partial B_\bullet$ of any maximizer $B_\bullet$ of $(\star)$ is C^1-smooth, thereby finding that if $|\nabla u_{B_\bullet}|$ is constant on $\partial B_\bullet$ then $B_\bullet$ is a Euclidean ball.