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Characterizing Rational Lemniscates: Is That Even Possible?

A polynomial lemniscate is a level curve of the modulus of a complex polynomial. As an application of the Riemann–Hurwitz formula, such a lemniscate is a Jordan curve if and only if all critical values of the polynomial lie inside the unit disk. A natural question is whether this characterization extends to rational lemniscates, the level curves of the modulus of rational functions. Unfortunately, the critical-value condition is only necessary.

In this talk, I will show that by shifting the perspective, from rational lemniscates themselves to asking which compact sets of the Riemann sphere arise as rational lemniscates, we obtain a complete characterization through three conditions that are topological, analytic, and algebraic.

This is joint work with Kirill Lazebnik and Malik Younsi.