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*A viscosity solution approach to the Feynman-Kac formula for a one-dimensional parabolic PDE with variable exponent coefficient*

This work establishes the existence and uniqueness of a solution for a one-dimensional parabolic Cauchy problem set on the positive half-line involving coefficients with variable exponent whose generator is associated with a stochastic differential equation involving state-dependent variable exponent. The problem is analyzed within the framework of viscosity solutions, addressing cases where classical solutions may not exist due to insufficient coefficient regularity. We demonstrate that the unique viscosity solution is given by the Feynman-Kac formula, thereby establishing a rigorous link between the probabilistic representation and the analytical solution. A key element of the proof relies on the property that the associated stochastic process remains strictly positive on its state space  $(0, \infty)$ , which allows for the application of local ellipticity arguments despite potential degeneracy at the boundary. The analysis is completed by applying the standard parabolic regularity theory to show that the viscosity solution possesses local Sobolev regularity in  $W_{m,loc}^{2,1}$ .

**Keywords.** stochastic process; viscosity solutions; Feynman-Kac formula; degenerate parabolic PDE; the comparison principle; the dynamic programming principle; local ellipticity.