Progress in differential equations and their applications in mathematical biology
Progrès dans le domaine des équations différentielles et leurs applications en biologie mathématique
(Org: **Elena Braverman** (University of Calgary), **Kunquan Lan** (Toronto Metropolitan University) and/et **Gail Wolkowicz**
(McMaster University))

MARYAM BASIRI, Toronto Metropolitan University

ELENA BRAVERMAN, University of Calgary

SUE ANN CAMPBELL, University of Waterloo
The Eigenvalue Spectrum of Distributed Delay Differential Equations with Large Mean Delay

This talk studies the eigenvalue spectrum of linear delay differential equations with a uniformly distributed delay kernel. We carry out asymptotic analysis in the limit of large mean delay to show that the spectrum splits into (i) a strong critical spectrum referring to a finite set of isolated, pure imaginary eigenvalues that are unaffected by delay, (ii) an asymptotic strong spectrum consisting of a finite set of eigenvalues with limits that are determined by non-delayed terms in the model, and (iii) a pseudo-continuous spectrum consisting of infinitely many eigenvalues that limit on the imaginary axis, with real parts that scale linearly with the delay. This behaviour is similar to the fixed delay case, but the distributed delay introduces additional spectral features, including a countably infinite number of horizontal asymptotes in the pseudo-continuous spectrum at frequencies inversely proportional to the width of the distribution. We validate our theoretical results through numerical studies of several examples and compare our findings with fixed-delay results from the literature. Finally, we apply the results to study the stability and bifurcations of a Wilson-Cowan model with a delayed self-coupling, large mean delay, and homeostatic plasticity. This is joint work with Isam Al-Darabsah and Bootan Rahman

HERMANN EBERL, University of Guelph
Travelling Waves in a highly degenerate PDE-ODE coupled model of cellulosic biofilm formation

In Eberl et al, *Biochem.Eng.J.*, 122, 2017, we presented a PDE-ODE coupled model describing inverse colony formation of cellulolytic biofilms which play a role in sustainable energy production, more specifically bioethanol that is produced from non edible, non-feedstock biomass such as wood and grass. The PDE component is a highly degenerated quasilinear diffusion reaction equation that encompasses both a porous-medium degeneracy where the dependent variable vanishes and a super-diffusion singularity as it approaches its maximum physical limit. In that earlier paper we also presented numerical simulations that suggest the possibility of travelling waves (for which there is also some, albeit scant, experimental evidence. In this talk I will present an existence and a non-existence theorem for such TW solutions, in dependence on model parameters.

CHRIS GOODRICH, UNSW Sydney, Austrailia

CHRISTOPHER HEGGERUD, University of Manitoba
Regime shifts in biology and tools to predict them

Regime shifts pose unique challenges when dealing with predictions and management of biological systems yet little headway has been made on understanding when a system might be in a transient state, or if a regime shift is imminent. In particular,

given a timeseries, it is difficult to determine the underlying mechanism causing a regime shift, or if one is occurring at all. Through a series of simplifications, we analyze synthetic data known to exhibit crawl-by type transient dynamics or that undergo some nonlinear excursion through state space that appears as a transient dynamic. Using dynamical systems theory, we create metrics that predict transient dynamics and furthermore to understand useful characteristics of the regime shift. These new metrics are additionally compared to typical early warning signals in ecology and the utility of both are discussed

KUNQUAN LAN, Toronto Metropolitan University

JENNIFER LAWSON, University of Calgary

CHENKUAN LI, Brandon University
Existence, Uniqueness, and Hyers–Ulam's Stability of the Nonlinear Bagley–Torvik Equation with Functional Initial Conditions

The nonlinear Bagley–Torvik equation is of fundamental importance, as it captures a realistic and intricate interplay among memory effects, nonlinearity, and functional dependence—making it a powerful model for a wide range of natural and engineered systems. Its analysis contributes significantly to both the theoretical development of fractional differential equations and their practical applications across science and technology. In this paper, we employ the inverse operator method, the multivariate Mittag-Leffler function, and several classical fixed-point theorems to establish sufficient conditions for the existence, uniqueness, and Hyers–Ulam stability of solutions to the nonlinear Bagley–Torvik equation with functional initial conditions. Finally, we present several examples by explicitly computing values of the multivariate Mittag-Leffler functions to illustrate the main results.

CHONGMING LI, Queen's University
Uniform Persistence Analysis of the Bacteria Persister Model

In this talk, we will explore eigenvalue problems arising from nonlocal elliptic equations that model the population dynamics of a bacterial group with presenting the persisters and available resource. The nonlocal component originates from epigenetic inheritance, more broadly referred to as a birth-jump process. In this setting, an offspring may leave its parent's location immediately after birth. In our model, this mechanism captures the phenotypic variations between parents and their offspring. Our main interest lies in understanding how changes in available resources—such as nutrient effects that constrain bacterial growth—impact the overall population size, especially in the presence of persister cells that cannot be eradicated. To address this, we establish a link between the principal eigenvalue and changes in resource availability, and then apply the super solution and maximum principle technique to examine the continuity and monotonicity of these effects.

XINZHI LIU, University of Waterloo
Observer-Based Adaptive Robust Control of Dual-Layer Multiagent Epidemic Models

This work proposes an innovative dual-layer multi-agent-based SIS epidemic model that integrates a physical contact layer and an information layer. The physical layer captures disease transmission through travel or migration between cities, while the information layer enables the exchange of infection data among healthcare providers across cities, even in the absence of direct physical connections. An observer is designed to estimate the infected fraction in each city by utilizing information from neighboring cities connected through the physical layer in a distributed manner. These estimates are subsequently used in the information layer to synchronize each city's infection trajectory with that of a virtual leader. Furthermore, the control input, typically defined in multi-agent systems, is employed as the sliding surface, whose stability is proven via Lyapunov analysis within the dual-layer SIS framework. To handle parameter uncertainties and ensure convergence to the sliding surface, an

adaptive sliding mode control strategy is developed, effectively integrating the dynamics of the physical and information layers to drive the system toward disease eradication. This is a joint work with Zohreh Abbasi.

CHUNHUA OU, Memorial University of Newfoundland

SUMAIRA REHMAN, Toronto Metropolitan University

ANDRÉ RICKES, University of Calgary

GUSTAVO CICCHINI SANTOS, Toronto Metropolitan University

ZHISHENG SHUAI, University of Central Florida, Orlando, FL, USA
A Tale of Two Incidence Functions: How Post-Infection Effects Shape Disease Dynamics

In epidemiological modeling, mass-action and standard incidence are two fundamental formulations of disease transmission. The former assumes a constant per-capita contact rate that scales with population size, while the latter accounts for limited contacts by normalizing the transmission term. Although both often produce similar long-term dynamics, their differences become pronounced when complex biological mechanisms are included.

In this study, we analyze a compartmental model incorporating post-infection mortality and partial immunity to compare these two incidence forms. For the mass-action model, bifurcation analysis reveals possible periodic outbreaks under certain parameter regimes, whereas the standard incidence model tends to suppress oscillations, leading to stable endemic equilibria. When infections persist, both analytical and numerical results show that endemic levels can remain low before rising sharply as transmission increases. These results highlight how incidence structure and reinfection jointly shape disease dynamics and have important implications for modeling long-term pathogen persistence in host populations.

AFRODITI TALIDOU, University of Calgary
Stability of front-like solutions of the FitzHugh-Nagumo equations on warped cylinders

The FitzHugh-Nagumo model is a reaction-diffusion system that describes the behavior of spiking neurons. While stability of traveling wave solutions is well understood in one dimension, much less is known in two dimensions. In this talk, we will examine the stability of traveling front solutions on the surfaces of both standard and warped cylinders. A standard cylinder has a constant radius, whereas the radius of a warped cylinder varies along its length. The latter geometry offers a more realistic model of the morphology of neuronal axons. I will outline how surface geometry enters the FitzHugh-Nagumo dynamics, describe criteria relevant to nonlinear stability, and illustrate geometric effects through numerical examples.

VITALI VOUGALTER, University of Toronto
Existence of stationary solutions for some integro-differential equations with the double scale anomalous diffusion

The work is devoted to the investigation of the solvability of an integro-differential equation in the case of the double scale anomalous diffusion with a sum of two negative Laplacians in different fractional powers in R^3. The proof of the existence of

solutions relies on a fixed point technique. Solvability conditions for the elliptic operators without the Fredholm property in unbounded domains are used.

LIN WANG, University of New Brunswick

TIANXU WANG, University of Alberta
Existence and asymptotic stability of a generic Lotka-Volterra system with nonlinear spatially heterogenous cross-diffusion

This article considers a class of Lotka-Volterra systems with multiple nonlinear cross-diffusion, commonly known as prey-taxis models. The existence and stability of classic solutions for such systems with spatially homogeneous sources and taxis have been studied in one- or two-dimensional space, however, the proof is non-trivial for a more general setting with spatially heterogeneous predation functions and taxis coefficient functions in arbitrary dimensions. This study introduces a new weighted L_ϵ^p-norm and extends some classical inequalities within this normed space. Coupled energy estimates are employed to establish initial bounds, followed by applying heat kernel properties and an advanced bootstrap process to enhance solution regularity. For stability analysis, we extend LaSalle's invariance principle to a general L^∞ setting and utilize it alongside Lyapunov functions to analyze the stability of each possible constant equilibrium. All results are achieved without introducing an extra logistic growth term for predators or imposing smallness conditions on taxis coefficients.

GAIL WOLKOWICZ, McMaster University
A predator-prey model with delay in both the prey and the predator growth terms

A predator-prey model with a discrete delay in both the prey and predator growth terms is formulated and analyzed. Delay is incorporated so that only those that survive the delay period, consistent with the decline terms in the model, can contribute to growth. The model without delay allows only convergence to a globally asymptotically stable equilibrium. Using delay in the predator growth term as a bifurcation parameter can result in complex dynamics, including a period doubling route to chaos followed by period halving, and eventual extinction of the predator population. Delay in the growth term of the prey on the other hand is shown to tame the dynamics.

JIANHONG WU, York University

HILAIRE EPSTEIN NONHOU ZOGO, Queen's University
Event-Triggered Control for an SIS Epidemic Model

In this talk, we will explore two control strategies, namely event-triggered feedback control and event-triggered impulse control, applied to an SIS epidemic model. In the event-triggered feedback control case, we construct a threshold to ensure the asymptotic stability of the disease-free equilibrium while maintaining the positivity of the proportion of infected individuals at all times. The control updates are triggered once the error between the proportion of infected individuals at the latest triggering time and the current proportion of infected individuals reaches that threshold. The update remains active and constant for a certain period of time before the next triggering instant is determined, and this control strategy operates continuously. The event-triggered impulse control case on the other hand, relies on a predefined threshold, aiming primarily at the convergence of the proportion of infected individuals toward the disease-free equilibrium. Control interventions are discrete and determined when the proportion of infected individuals reaches that threshold. Additionally, we analyze the effect of execution delays in both control strategies and demonstrate that in either control case, the controlled SIS system does not exhibit Zeno behavior.

XINGFU ZOU, Western University