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A tale on trascendence and arithmetic equivalence

This talk is about current work in progress that is inspired by TeamsUp! project (25frg504). It is well known that the Dedekind zeta function determines the product of the class number and the regulator of a number field, but not each of these invariants individually. As Perlis showed in the 1970s, there exist number fields with the same Dedekind zeta function but with different regulators and class numbers.

In this talk, we explore a converse phenomenon. Our main result is that, assuming the algebraic independence of logarithms (the weak Schanuel conjecture), two totally real number fields with the same regulator must have the same Dedekind zeta function. As a further consequence of our methods, we show that the weak Schanuel's conjecture implies that the residues at $s = 1$ of two distinct Dedekind zeta functions of totally real number fields are linearly independent over the algebraic closure of $\mathbb{Q}$ in $\mathbb{C}$.

It is worth mentioning that Gun, Murty, and Rath previously proved that the weak Schanuel conjecture implies the transcendence of these residues for arbitrary number fields, so this may be viewed as a very tiny addendum to that story.