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Universal Nonlinear Learning of High-Dimensional Anisotropic Sobolev Functions from Point Samples

A central problem in scientific machine learning is high-dimensional function recovery from limited data. Widths, an important concept from information-based complexity, provide a standard way to quantify this. At the same time, universal approximation theorems highlight the representational power of nonlinear models such as neural networks and have become a central theme in machine learning. In anisotropic settings, where different coordinates exhibit different smoothness, universality questions arise naturally. In this work, we study them for recovery from point samples (standard information) in periodic Sobolev spaces with anisotropic smoothness, including both anisotropic Sobolev and anisotropic mixed-smoothness Sobolev spaces. Our approach is a theoretical nonlinear reconstruction scheme inspired by compressed sensing, for which we derive worst-case upper bounds on the recovery error. We prove a universal approximation result by showing that our nonlinear reconstruction map achieves asymptotically better error guarantees, simultaneously over a family of anisotropic smoothness classes, than any linear method, thus justifying nonlinear algorithms for universal approximation in such spaces. In terms of widths, we establish matching upper and lower bounds for nonlinear widths, thereby identifying the optimal performance of sampling-based recovery on these spaces. Such results on nonlinear widths in anisotropic settings have been largely absent, so our bounds are of concrete interest for sampling recovery and their learning-theoretic applications.