
Combinatorial Design Theory
Théorie de la conception combinatoire

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A Complete Solution to the Generalized HOP with One Round Table

The Honeymoon Oberwolfach Problem (HOP), introduced by Šajna, is a recent variant of the classic Oberwolfach Problem. This problem asks whether it is possible to seat $2m_1 + 2m_2 + \dots + 2m_t = 2n$ participants, consisting of n newlywed couples, at t round tables of sizes $2m_1, 2m_2, \dots, 2m_t$ for $2n - 2$ successive nights, so that each participant sits next to their spouse every night and next to every other participant exactly once. This problem is denoted by $\text{HOP}(2m_1, 2m_2, \dots, 2m_t)$. Jerade, Lepine, and Šajna have studied the HOP and resolved several important cases.

We generalized the HOP by allowing tables of size two, relaxing the previous restriction that tables must have a minimum size of four. In the generalized HOP, we aim to seat the $2n$ participants at s tables of size 2 and t round tables of sizes $2m_1, 2m_2, \dots, 2m_t$, where $2n = 2s + 2m_1 + 2m_2 + \dots + 2m_t$ and $m_i \geq 2$, while preserving the adjacency conditions of the HOP. We denote this problem by $\text{HOP}(2^{(s)}, 2m_1, \dots, 2m_t)$.

In this talk, we present a general approach to this problem and provide a solution to the generalized HOP with one round table, showing that the necessary condition for $\text{HOP}(2^{(s)}, 2m)$ to have a solution is also sufficient.

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Cyclic partitions of complete hypergraphs and large sets of combinatorial designs

We consider cyclic partitions of the complete k -uniform hypergraph on a finite set V , and their relationship to combinatorial designs. A t -complementary k -hypergraph is a k -uniform hypergraph with vertex set V and edge set E for which there exists a permutation $\theta \in \text{Sym}(V)$ such that the sets $E, E^\theta, E^{\theta^2}, \dots, E^{\theta^{t-1}}$ partition the set of all k -element subsets of V . Such a permutation θ is called a (t, k) -complementing permutation. The t -complementary k -hypergraphs are a natural generalization of the almost self-complementary graphs, since the associated (t, k) -complementing permutation θ decomposes the complete k -uniform hypergraph into t isomorphic k -hypergraphs, which are permuted cyclically by θ . When these t -complementary k -hypergraphs in the decomposition are also *regular*, then they form a large set of t isomorphic combinatorial designs. We

will look at some algebraic constructions for large sets of combinatorial designs that arise from these cyclic decompositions, including one which generalizes the well know Paley graph construction.

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CALEB JONES, Toronto Metropolitan University

WILLIAM KELLOUGH, Memorial University of Newfoundland
BIBDs That Almost Have Locally Equitable Colourings

In this talk, we analyze ℓ -colourings of (v, k, λ) -BIBDs where within each block, one colour is absent and the rest appear $\frac{k}{\ell-1}$ times. We give necessary conditions for such colourings to exist. We show how Hadamard matrices, affine planes, and twin prime powers can be used to construct such coloured BIBDs.

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Constructing uniquely 2-colourable 4-cycle decompositions

A cycle system of order n is a decomposition of the edges of the complete graph K_n into cycles of a fixed length. A cycle system is said to be k -colourable if we can assign k colours to its vertices so that no cycle is monochromatic. If a cycle system is k -colourable but not $(k-1)$ -colourable, it is called k -chromatic. A k -colourable cycle system is uniquely k -colourable if its colouring is unique up to the permutation of colour classes.

The study of colouring cycle systems has been explored in various settings. In particular, Horsley and Pike have examined the existence of k -chromatic m -cycle systems for any integers $m > 2$ and $k > 1$. While Forbes has investigated 3-cycle systems with unique 3-colourability, the existence of uniquely k -colourable m -cycle systems in general remains an open problem.

In this talk, we mainly focus on the construction of an infinite family of uniquely 2-colourable 4-cycle systems and also a uniquely 2-colourable 4-cycle decomposition of $K_n - I$, for infinitely many integers $n \geq 2$. These constructions contribute to the broader study of uniquely colourable cycle systems and open new directions for future research.

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