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Factorization of finite groups

Let $(G, \cdot)$ be a finite multiplicative group with identity e . For $A, B \subseteq G$, define

$$AB = \{gh : g \in A, h \in B\}$$

and note that AB is a multi-set. We say that (A, B) is a (λ, μ) -factorization of G if in the product AB each non-identity element appears λ times and the identity occurs μ times. In the group ring $\mathbb{Z}[G]$ we write

$$AB = \lambda(|G| - e) + \mu e$$

Given a subset $A \subseteq G$, if $B \subseteq G$ satisfies this group ring equation, then we say that B is a (λ, μ) -mate of A . A λ -mate with $\mu = 0$ and is simply called a λ -mate and if $\lambda = 1$ and $\mu = 0$, then it is called a mate.

A $(1, 0)$ -factorization of G is called a near-factorization of G and is where my story begins. However a $(1, 1)$ -factorization AB of G when neither A nor B are subgroups of G has perhaps received the most attention by investigators and will likely be where my story will end. Between these two events we have explored factorizations when $\lambda > 1$.

If there is a (λ, μ) -factorization (A, B) of G , with $\lambda \neq \mu$, then there is an explicit formula for B in terms of A . This leads to a direct method for computing a (λ, μ) -mates when they exist. Surprisingly it appears that it is more efficient to compute (λ, μ) -mates using sophisticated backtracking tools.