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Simplicial Resolutions of Powers of Monomial Ideals

Given a monomial ideal I minimally generated by q monomials, we construct a simplicial complex $\mathbb{M}_q^2$ that supports a free resolution of the square I^2. We also define a natural subcomplex $\mathbb{M}^2(I)$, depending on the specific generators of I, that likewise supports a resolution of I^2. Our framework yields new bounds on the projective dimension of second powers of monomial ideals and provides sharper bounds for the Betti numbers of I^2 compared to those obtained from the Taylor resolution.

Furthermore, we introduce permutation ideal $\mathcal{T}_q$, generated by q monomials, and prove that for any monomial ideal I with q generators, $\beta(I^2) \leq \beta(\mathcal{T}_q^2)$. We also show that the simplicial complex $\mathbb{M}_q^2$ supports the minimal resolution of $\mathcal{T}_q^2$, and that $\mathbb{M}_q^2$ in fact coincides with the Scarf complex of $\mathcal{T}_q^2$. Finally, we present analogous constructions and results for third and higher powers of general monomial ideals. This is joint work with Susan Cooper and Sara Faridi, with additional related work in progress with Sara Faridi and Chau Trung.