AMAURY DE BURGOS, University of Calgary
The length of cyclic algebras

Cyclic algebras, introduced by Leonard Eugene Dickson in 1906, were one of the earliest examples of non-commutative division algebras over a field. A notable numerical invariant of any algebra over a field is its length, defined as the length of its longest chain of linear subspaces. In 2016, the length of 4-dimensional cyclic algebras was proven to be 2 (Guterman & Kudryavtsev). More recently, in 2021, the length of cyclic algebras of dimension 9, 16, and 25 was stated to be 4, 6, and 8 respectively (Miguel). Through the use of two counterexamples, we show these latter values are ill-derived, meaning the length of cyclic algebras of dimension greater than 4 is still an open problem. In pursuit of solving this open problem, we construct an infinite family of cyclic division algebras and give a lower bound on the length of its members.

XINWEN DING, University of Toronto
Walk-on-Interfaces: A Monte Carlo Estimator for Elliptic Interface Problem

Elliptic interface problems arise in many areas of science and engineering, modeling heterogeneous materials whose physical properties change abruptly across internal boundaries. Computing solutions to these problems efficiently and accurately remains challenging, especially in domains with multiple irregular interfaces. In this poster, we present Walk-on-Interfaces (WoI), a grid-free Monte Carlo estimator for Neumann elliptic interface problems with general flux jump conditions. Unlike many numerical schemes, WoI maintains uniform accuracy throughout the domain and avoids near-interface singularities. Moreover, gradients of the solution can be estimated at almost no additional cost by differentiating the Green's function within WoI. Taking a scientific machine learning approach, we train a deep neural network to filter out high-frequency sampling noise, yielding a smooth and continuous representation of the solution. The resulting method is highly parallelizable, scales naturally to high dimensions, and can solve problems that are intractable for traditional numerical solvers. Numerical experiments demonstrate the effectiveness of the approach and highlight its potential for real-world applications.

CAMERON JAKUB, University of Guelph
Depth Degeneracy in Neural Networks: Vanishing Angles in Fully Connected ReLU Networks on Initialization

Despite remarkable performance on a variety of tasks, many properties of deep neural networks are not yet theoretically understood. One such mystery is the depth degeneracy phenomenon: the deeper you make your network, the closer your network is to a constant function on initialization. In this paper, we examine the evolution of the angle between two inputs to a ReLU neural network as a function of the number of layers. By using combinatorial expansions, we find precise formulas for how fast this angle goes to zero as depth increases. These formulas capture microscopic fluctuations that are not visible in the popular framework of infinite width limits, and leads to qualitatively different predictions. We validate our theoretical results with Monte Carlo experiments and show that our results accurately approximate finite network behaviour. We also empirically investigate how the depth degeneracy phenomenon can negatively impact training of real networks. The formulas are given in terms of the mixed moments of correlated Gaussians passed through the ReLU function. We also find a surprising combinatorial connection between these mixed moments and the Bessel numbers that allows us to explicitly evaluate these moments.

KENNETH SHEN, Carleton University Math Enrichment Centre
Families of rational-sided triangles with the same area and perimeter

Can two non-congruent triangles share the same area and perimeter? While it may seem impossible at first, we soon find that for any non-equilateral triangle with *real*-valued sides, there exist infinitely many other triangles with the same area and perimeter. This raises the following question: what would an analogous result look like for *rational*-sided triangles?

We introduce the idea of *confined* triangles: rational-sided triangles for which only finitely many other non-congruent rational-sided triangles share the same area and perimeter. A triangle is *isolated* if no such companions exist. We prove that confined *scalene* triangles must be isolated, and completely characterize the structure of confined triangles, showing that only three configurations are possible: a single isolated isosceles triangle, exactly two isosceles triangles, or a single isolated scalene triangle.

Finally, we prove that the asymptotic proportion of confined triangles that are scalene tends to zero.

XUEMENG WANG, Simon Fraser University
Christoffel Adaptive Sampling for Sparse Random Feature Expansions

Random feature models are powerful tools for approximating high-dimensional functions and solving PDEs. Sparse random feature expansion (SRFE) enhances these methods by incorporating sparsity and compressive sensing, which is especially beneficial in data-scarce settings. We integrate active learning with SRFE by using the Christoffel function to guide an adaptive sampling process, dynamically selecting informative samples. Drawing random samples via the Christoffel function allows a weighted least-squares approximation with near-optimal sample complexity. Numerical experiments show that Christoffel adaptive sampling maintains high accuracy, demonstrating strong potential for scientific computing.