
KENNETH SHEN, Carleton University Math Enrichment Centre
Families of rational-sided triangles with the same area and perimeter

Can two non-congruent triangles share the same area and perimeter? While it may seem impossible at first, we soon find that for any non-equilateral triangle with *real*-valued sides, there exist infinitely many other triangles with the same area and perimeter. This raises the following question: what would an analogous result look like for *rational*-sided triangles?

We introduce the idea of *confined* triangles: rational-sided triangles for which only finitely many other non-congruent rational-sided triangles share the same area and perimeter. A triangle is *isolated* if no such companions exist. We prove that confined *scalene* triangles must be isolated, and completely characterize the structure of confined triangles, showing that only three configurations are possible: a single isolated isosceles triangle, exactly two isosceles triangles, or a single isolated scalene triangle.

Finally, we prove that the asymptotic proportion of confined triangles that are scalene tends to zero.