
GRACE D'AGOSTINO, University of Guelph
Uncertainty Analysis of a River Quality Model

Uncertainty analyses were performed on a simple river quality model, the Streeter-Phelps equations, using parameter estimates, comparison theorems, and differential inequalities.

Model parameters in environmental systems are often non-constant and may only be known with some uncertainty. When parameters are not known exactly, the solution of the model cannot be obtained analytically or numerically.

The Streeter-Phelps equations are two partial differential equations describing the transport of pollutants, also called the biological oxygen demand (BOD), and dissolved oxygen in a river. In their original formulation, the model permits negative dissolved oxygen concentrations due to the lack of dependence of BOD degradation on available oxygen, marking a breakdown of the model. To remedy this, a dependence was imposed using the Monod function, creating a situation-adaptive transition of solutions between when oxygen is abundant and when oxygen is limited.

To satisfy the quasi-monotonicity condition of the Comparison Theorem for Quasimonotone Increasing Systems, the system was redefined in terms of the oxygen deficit, which is the difference in the saturation concentration and the dissolved oxygen concentration. Possibilistic regions in which the solution of the model with unknown parameters exist were constructed.

Parameter estimates were chosen arbitrarily based on data reported by the Grand River Conservation Authority for illustrative purposes. Plots were produced and limitations were discussed.