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Toric Compactifications and Critical Points at Infinity in Analytic Combinatorics

The field of Analytic Combinatorics in Several Variables (ACSV) provides powerful tools for deriving asymptotic information from multivariate generating functions. A key challenge arises when standard saddle-point techniques fail due to the presence of critical points at infinity (CPAI), obstructing local analyses near singularities. Recent work has shown that Morse-theoretic decompositions remain valid under the absence of CPAI, traditionally verified using projective compactifications. We present a toric approach to compactification that leverages the Newton polytope of a generating function to construct a toric variety tailored to the function's combinatorial structure. This refinement not only tightens classification of CPAI but also enhances computational efficiency. Through concrete examples and an introduction to tropical and toric techniques, we will demonstrate how these methods clarify the asymptotic landscape of ACSV problems, especially in combinatorially meaningful settings. This poster draws on joint work studying toric compactifications as a bridge between algebraic geometry and analytic combinatorics.