DENIS VINOKUROV, Université de Montréal

Topological Tensor Products, Harmonic Maps, and Spectral Optimization

In the classical scalar setting, the Rellich–Kondrachov theorem provides compact Sobolev embeddings that are central to many arguments in analysis. This compactness fails for Sobolev maps with values in infinite-dimensional targets, such as Hilbert spaces, and standard direct methods break down.

I will explain how techniques from the theory of topological tensor products allow one to recover a useful compactness framework for certain classes of variational integrals for vector-valued functions. As a key example, we consider Dirichlet-type energies whose critical points are harmonic maps into the infinite-dimensional Hilbert sphere. Their energy densities also arise as critical points of a Laplace eigenvalue optimization problem. We address both the existence of such optimal densities and the regularity of the associated harmonic maps.