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On asymptotic Lebesgue's universal covering problem

A classical theorem of Jung states that any set of diameter 1 in an n-dimensional Euclidean space is contained in a ball J_n of radius $\sqrt{\frac{n}{2n+2}}$; in other words, J_n is a universal cover in $\mathbb{E}^n$.

Lebesgue's universal covering problem, posed in 1914, asks for the convex set of smallest area that serves as a universal cover in the plane ($n = 2$). We show that in high dimensions, Jung's ball J_n is asymptotically optimal with respect to volume: for any universal cover $U \subset \mathbb{E}^n$,

$$\mathrm{Vol}(U) \geq (1 - o(1))^n \mathrm{Vol}(J_n).$$

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