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*Nonabelian Sidon sets and extremal problems on digraphs*

An  $S_k$ -set is a subset of a group whose  $k$ -tuples have distinct products. We give explicit constructions of large  $S_k$ -sets in the groups  $\text{Sym}(n)$  and  $\text{Alt}(n)$  and of large  $S_2$ -sets in  $\text{Sym}(n) \times \text{Sym}(n)$  and  $\text{Alt}(n) \times \text{Alt}(n)$ , as well as some probabilistic constructions for 'nice' groups. We show that if  $k$  is even and  $\Gamma$  has a normal abelian subgroup with bounded index then any  $S_k$ -set has size at most  $(1 - \varepsilon)|\Gamma|^{1/k}$ . The  $S_k$ -sets are related to the following graph-theoretic problem: determine the largest possible minimum outdegree in a directed graph with no subgraph in  $\{C_{2,2}, \dots, C_{k,k}\}$ , where  $C_{\ell,\ell}$  is the orientation of  $C_{2\ell}$  with two maximal directed  $\ell$ -paths. Contrasting with undirected cycles, the extremal minimum outdegree for  $\{C_{2,2}, \dots, C_{k,k}\}$  is much smaller than that for any  $C_{\ell,\ell}$ . We count the directed Hamilton cycles in one of our constructions to improve the upper bound for a problem on Hamilton paths posed by Cohen, Fachini, and Körner. Joint work with Michael Tait.