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A Nordhaus–Gaddum Problem for the Spectral Gap of a Graph

We study how quickly a random walk on a graph mixes by examining the spectral gap of its transition probability matrix. For any graph G on n vertices and its complement $\overline{G}$, we prove that

$$\max\{\mathrm{gap}(G),\, \mathrm{gap}(\overline{G})\} = \Omega(1/n).$$

When both the minimum and maximum degrees of G are $\Omega(n)$, this maximum spectral gap improves to $\Theta(1)$. We also establish lower bounds of order $\Omega(1/n)$ when the maximum degree is $n - O(1)$, or when G is the join of two graphs.

In contrast, we construct families of connected graphs whose complements are also connected for which

$$\max\{\mathrm{gap}(G),\, \mathrm{gap}(\overline{G})\} = O(n^{-3/4}).$$

These results illustrate how complementary graph structures constrain spectral-gap behaviour.