DAVID GRYNKIEWICZ, University of Memphis

Towards a Kneser-Pollard Theorem

Let $A, B \subset G$ be finite, nonempty subsets of an abelian group G. For $t \geq 1$, the t-popular sumset $A +_t B$ denotes all $x \in G$ having t distinct tuples $(a_1, b_1), \ldots, (a_t, b_t) \in A \times B$ with $x = a_1 + b_1 = \ldots = a_t + b_t$. When $t = 1$, Kneser's Theorem says

$$|A +_1 B| \geq |A| + |B| - |H|,$$

where $H = \{x \in G : \ x + A + B = A + B\} \leq G$ is the stabilizer of $A + B$. When $|G| = p$ is prime, Pollard's Theorem says

$$\sum_{i=1}^{t} |A +_i B| \geq t \min\{p, |A| + |B| - t\}.$$

When $|G| = p$ and $t = 1$, both results coincide. It is an open question to give a Kneser-type generalization of Pollard's Theorem to a general abelian group G. The best partial result is a theorem that describes the structure of A and B when

$$\sum_{i=1}^{t} |A +_i B| < t|A| + t|B| - (2t^2 - 3t + 2),$$

showing there must exist $A' \subseteq A$ and $B' \subseteq B$ with $|A \setminus A'| + |B \setminus B'| \leq t - 1, \ \ A' +_t B = A' + B' = A +_t B$, and

$$\sum_{i=1}^{t} |A +_i B| \geq t(|A| + |B| - |H|),$$

where H is the stabilizer of $A' + B' = A +_t B$. These conclusions, combined with classical sumset results, imply a strong structural description of A and B. However, the term $2t^2 - 3t + 2$ is too large for this result to encompass a full generalization of both Kneser and Pollard's Theorems, which would require a result valid using t^2 rather than $2t^2 - 3t + 2$. Here we achieve progress by improving the main quadratic term in the bound from $2t^2$ to $\frac{4}{3}t^2$. Joint work with Runze Wang.