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Waring's problem and its relatives

In 1770, E. Waring made an assertion these days interpreted as conjecturing that when k is a natural number, all positive integers may be written as the sum of a number g(k) of positive integral k-th powers, with g(k) finite. This conjecture generalises the familiar theorem of Lagrange showing that all positive integers are the sum of four squares. Since the work of Hardy and Littlewood a century ago, attention has largely shifted to the problem of bounding G(k), the least number s having the property that all sufficiently large integers can be written as the sum of s positive integral k-th powers. The principal tool for investigations associated with Waring's problem, namely the Hardy-Littlewood (circle) method, is Fourier-analytic in flavour. In this talk we survey progress on Waring's problem that has emerged in the past 15 years, its connection with numbers having only small prime divisors as well as recent progress on discrete harmonic analysis, and implications of these recent ideas for cognate problems. Emerging developments, for example, touch on the topic of representing prime numbers as sums of powers, and impact the apparently mundane topic of controlling small solutions of congruences.