Asymptotic Geometric Analysis
Analyse géométrique asymptotique

(Org: **Susanna Dann** (Universidad de los Andes, Bogota, Colombia), **Daniel Galicer** (Universidad de Buenos Aires , Argentina) and/et **Alexander Litvak** (University of Alberta))

ANDRII ARMAN, University of Manitoba
Bodies of constant width that have small volume

Oded Schramm (1988) asked if there are convex bodies (in $\mathbb{R}^n$) of constant width 2 with the volume that is exponentially smaller than the volume of the unit ball $\mathbb{B}^n$.

In this talk I will provide a construction that answers the question of Schramm in affirmative, namely I will show that for a large enough n there is a convex body $M_n \subset \mathbb{R}^n$ of constant width 2 such that $\text{Vol}(M_n) \leq 0.9^n \text{Vol}(\mathbb{B}^n)$.

This talk is based on a joint work with Andriy Bondarenko, Fedor Nazarov, Andriy Prymak, and Danylo Radchenko.

VISHESH JAIN, University of Illinois Chicago
Entangled states are typically incomparable

Consider a bipartite quantum system, where Alice and Bob jointly possess a pure state $|\psi\rangle$. Using local quantum operations on their respective subsystems, and unlimited classical communication, Alice and Bob may be able to transform $|\psi\rangle$ into another state $|\phi\rangle$. Famously, Nielsen's theorem provides a necessary and sufficient algebraic criterion for such a transformation to be possible (namely, the entanglement spectrum of $|\phi\rangle$ should majorise the entanglement spectrum of $|\psi\rangle$). In the same paper, Nielsen conjectured that in the limit of large dimensionality, for almost all pairs of states $|\psi\rangle, |\phi\rangle$ (according to the natural unitary invariant measure) such a transformation is not possible. That is to say, typical pairs of quantum states $|\psi\rangle, |\phi\rangle$ are entangled in fundamentally different ways, that cannot be converted to each other via local operations and classical communication.

Via Nielsen's theorem, this conjecture can be equivalently stated as a conjecture about majorisation of spectra of random matrices from the so-called trace-normalised complex Wishart–Laguerre ensemble. Concretely, let X and Y be independent $n \times m$ random matrices whose entries are i.i.d. standard complex Gaussians; then Nielsen's conjecture says that the probability that the spectrum of $XX^\dagger/\text{tr}(XX^\dagger)$ majorises the spectrum of $YY^\dagger/\text{tr}(YY^\dagger)$ tends to zero as both n and m grow large. We prove this conjecture, and we also confirm some related predictions of Cunden, Facchi, Florio and Gramegna.

Joint work with Matthew Kwan (IST Austria) and Marcus Michelen (UIC).

KIRILL KASHKAN, University of Toronto
Dense Forests With Low Visibility

A set of points F in $\mathbb{R}^d$ is called a dense forest if there is a decreasing visibility function $V(\varepsilon)$ such that any line segment in $\mathbb{R}^d$ of length $V(\varepsilon)$ has a point of F within distance ε of it and F has finite density. Since being introduced in the 2010s, many forests with desirable properties have been constructed. Those properties being: low visibility, the forest being uniformly discrete, or having a deterministic construction.

This talk will present a dense forest constructed by modifying a set of points obtained from a Poisson Process. The dense forest has visibility $V(\varepsilon) \in O(\varepsilon^{-(d-1)} \log \varepsilon^{-1})$.

SERHII MYROSHNYCHENKO, University of the Fraser Valley
Stability of simplex slicing

We establish dimension-free stability estimates for volume of central hyperplane sections of the regular simplex. This provides a refinement of Webb's sharp upper bound on the volume of central slices from 1996. Incidentally, we investigate Lipschitzness of volume of central sections of arbitrary convex bodies. Joint work with C. Tang, K. Tatarko, T. Tkocz.

MATHIAS SONNLEITNER, University of Alberta
Strange shadows of ℓ_p-balls

With growing dimension, a typical random projection of the ℓ_p^n-ball onto a subspace of fixed dimension tends to a Euclidean ball of some fixed radius. This is related to the strong law of large numbers of the p^*-sum of independent and identically distributed line segments, where p^* is the conjugate index. It is thus not surprising that L_{p^*}-zonoids appear as shadows and the typical shadow of the ℓ_p^n-ball is close to the above Euclidean ball. We are interested in shadows which are strange, meaning that they occur with probability exponentially decaying with some rate. This is formalized by a large deviations principle in the space of convex bodies equipped with Hausdorff distance in the case of $p > 2$. Building on work of Kim and Ramanan, we identify the rate of decay via the entropy of representing measures of the corresponding L_{p^*}-zonoid. Via duality we obtain a result for random sections. Based on joint work with Zakhar Kabluchko.

ALINA STANCU, Concordia University
An asymmetric flow with many symmetric solutions

We define an isotropic, asymmetric, flow on smooth, compact, convex surfaces in Euclidean 3-space that exhibits distinct centrally-symmetric self-similar solutions including the Euclidean ball. The flow is not affine invariant, yet ellipsoids of revolution evolve self-similarly and can be generalized in all dimensions. This is joint work with Valentina-Mira Wheeler.

VEATRIKI ELENI VRITSIOU, University of Alberta
Illuminating certain high-dimensional 1-unconditional convex bodies

Let us think of a convex body in $\mathbb{R}^n$ as an opaque object, and let us place point light sources around it, wherever we want, to illuminate its entire surface. What is the minimum number of light sources that we need? The Hadwiger-Boltyanski illumination conjecture from the 1950's-60's states that we need at most 2^n light sources, with the upper bound conjectured to be attained only by parallelotopes.

The conjecture is still open in dimension 3 and above, and has only been fully settled for certain classes of convex bodies (e.g. zonoids, bodies of constant width, etc.). Moreover, there are some rare examples for which a basic, folklore argument could quickly lead to the upper bound 2^n, while at the same time understanding the equality cases has remained elusive for decades. One such example would be convex bodies very close to the cube, which was settled by Livshyts and Tikhomirov in 2017.

In this talk I will discuss another such instance, which comes from the class of 1-unconditional convex bodies, and which also 'forces' us to settle the conjecture for a few more cases of 1-unconditional bodies. This is based on joint work with Wen Rui Sun, and our arguments are primarily combinatorial.

SUDAN XING, University of Arkansas at Little Rock
On the s-Gaussian Measure in $\mathbb{R}^n$

In this talk, I will present my recent work with Prof. Youjiang Lin. The s-Gauss probability space is introduced based on the s-Gaussian density function in $\mathbb{R}^n$ for $s \geq 0$, a generalization of the classic Gaussian density function. We also propose the (s, k)-Ehrhard symmetrization which is an extension of the traditional Ehrhard symmetrization for sets in $\mathbb{R}^n$. In particular, we establish the s-Gaussian isoperimetric inequality with respect to s-Gaussian measure in $\mathbb{R}^2$ and prove the s-Ehrhard-Borell inequalities for $s > 0$ when one of the two sets is a Borel set whilst the other being a convex set as well as the case when two sets are convex in $\mathbb{R}^1$.