SCOTT WESLEY, Dalhousie University
Verifying and Simplifying Tietze Transformations

Invertible operations are described by groups. Many examples of groups, such as permutations and reversible circuits also admit nice combinatorial descriptions. These combinatorial descriptions often take the form of group presentations, which describe groups in terms of their generating elements, and the relations those elements satisfy. Unfortunately, identifying when two group presentations are isomorphic is known to be undecidable; that is, no algorithm can determine when arbitrary group presentations are isomorphic. In practice, people rely on Tietze transformations to build these isomorphisms by-hand. However, rigorously constructing proofs from Tietze transformations often proves to be tedious and time-consuming.

In this talk, we propose a new framework for building isomorphisms from Tietze transformations. We start by viewing Tietze transformations as a proof system for establishing group isomorphisms. From this perspective, the validity of a Tietze-style proof is decidable, whereas the existence of a Tietze-style proof is undecidable. On top of this foundation, we show that basic semantic arguments and compositional reasoning can be added to the proof system, without losing the decidability of proof validity. We then implement a software package, which we call Tietze, to automatically validate proofs in this proof system. We demonstrate the effectiveness of Tietze to solve real problems, and discuss directions for future work, such as proof visualization and interactive theorem proving.