Operator Theory, Function Theory, and Geometry: Connections to Corona Problems and Geometric Analysis
(Org: **Alexander Brudnyi** and/et **Mahishanka Withanachchi** (University of Calgary))

AKRAM ALDROUBI, Vanderbilt University
Dynamical sampling: source term recovery and frames

In this talk, I will address the problem of recovering a source terms in a discrete dynamical system represented by $x_{n+1} = Ax_n + w$, where x_n is the n-th state in a Hilbert space $\mathcal{H}$, A is a bounded linear operator in $\mathcal{B}(\mathcal{H})$, and w is a source term within a closed subspace W of $\mathcal{H}$. The focus is on the stable recovery of w using time-space sample measurements formed by inner products with vectors from a Bessel system $\mathcal{G} \subset \mathcal{H}$. These types of results may be relevant to applications such as environmental monitoring, where precise source identification is critical. This work is in collaboration with Rocio Diaz Martin Le Gong, Javad Mashreghi, and Ivan Medri.

ILIA BINDER, University of Toronto
Conformal Dimension of Planar fractals.

The *conformal dimension* of a set is the minimal Hausdorff dimension of its quasisymmetric image. In this talk, I will discuss the conformal dimensions of various planar fractals, including Bedford-McMullen sets and self-affine fractal percolation clusters. I will also demonstrate that the Brownian graph is *minimal*, meaning its conformal dimension is $3/2$, which is also its Hausdorff dimension.

This work is a collaboration with Hrant Hakobyan from Kansas State University and Wenbo Li from Peking University.

LUDOVICK BOUTHAT, Université Laval
Exploring Hadamard multipliers on weighted Dirichlet spaces through L-matrices

The Hadamard product of two power series is obtained by multiplying them coefficientwise. In 2020, Mashreghi and Ransford characterized those power series that act as Hadamard multipliers on all weighted Dirichlet spaces on the disk with superharmonic weight. These power series correspond to those whose associated L-matrix defines a bounded operator on ℓ^2. An L-matrix is an infinite matrix $\mathcal{L}$ whose entries are of the form $\mathcal{L}_{i,j} = a_{\max\{i,j\}}$ for some complex sequence $(a_n)_{n\geq 0}$. In this talk, we present several conditions on the sequence $(a_n)_{n\geq 0}$ for $\mathcal{L}$ to be a bounded operator on ℓ^2 and we present a particular set of L-matrices for which we are able to exactly determine the norm.

This work is a collaboration with Javad Mashreghi.

ALEX BRUDNYI, University of Calgary
Runge-Type Approximation Theorem for Banach-valued H^∞ Functions on a Polydisk

Let $\mathbb{D}^n \subset \mathbb{C}^n$ be the open unit polydisk, $K \subset \mathbb{D}^n$ be an n-ary Cartesian product of planar sets, and $\widehat{U} \subset \mathfrak{M}^n$ be an open neighbourhood of the closure $\bar{K}$ of K in $\mathfrak{M}^n$, where $\mathfrak{M}$ is the maximal ideal space of the algebra H^∞ of bounded holomorphic functions on $\mathbb{D}$. Let X be a complex Banach space and $H^\infty(V, X)$ be the space of bounded X-valued holomorphic functions on an open set $V \subset \mathbb{D}^n$. We show that any $f \in H^\infty(U, X)$, where $U = \widehat{U} \cap \mathbb{D}^n$, can be uniformly approximated on K by ratios h/b, where $h \in H^\infty(\mathbb{D}^n, X)$ and b is the product of interpolating Blaschke products such that $\inf_K |b| > 0$. Moreover, if $\bar{K}$ is contained in a compact holomorphically convex subset of $\widehat{U}$, then h/b above can be replaced by h for any f. The results follow from a new constructive Runge-type approximation theorem for Banach-valued holomorphic functions on open subsets of $\mathbb{D}$ and extend the fundamental results of Suarez on Runge-type approximation for analytic germs on compact subsets of $\mathfrak{M}$. They can also be applied to the long-standing corona problem which asks whether $\mathbb{D}^n$ is dense in the maximal ideal space of $H^\infty(\mathbb{D}^n)$ for all $n \geq 2$.

DAMIR KINZEBULATOV, Université Laval
Feller generators with singular drifts in the critical range

I will discuss recent progress on a long standing problem of describing admissible singular drifts of Brownian motion. The first part deals with a sharp result on the magnitude of drift singularities that separates well-posedness from a blow up. This requires us to work, not quite expectedly, in appropriate "critical" Orlicz space (a rather compelling instance of the Lp theory). Informally, it turned out that strengthening appropriately the topology of the space where the Kolmogorov backward equation is considered allows to handle stronger singularities of the drift. This leads to the second part of the talk (joint with Yu.A.Semenov) on the Feller semigroup and a detailed well-posedness theory of the corresponding martingale problem for the entire subcrticial range of the mangitudes of singularities of the drift. The proof uses in a crucial manner some operator-theoretric techniques, such as Trotter's approximation theorem.

PIERRE OLIVIER, University of Quebec in Three-Rivers
Divergence of Taylor Polynomials in de Branges-Rovnyak Spaces

In this talk, I will present sufficient conditions for the existence of a function in a given de Branges-Rovnyak space for which the Taylor series is unbounded in norm or diverges in norm. The result is a consequence of a refined version of the boundedness principle established by Müller and Vrsovsky.

This is a joint work with Thomas Ransford.

ERIC SAWYER, McMaster University
Probabilistic and Deterministic Fourier Extension

We discuss the proof of the probabilistic Fourier extension theorem, and possible applications to the deterministic conjecture.

KRYSTAL TAYLOR, Ohio State University
Efficient Coverings of Fractal sets by curves

A classic theorem of Davies states that a set of positive Lebesgue measure can be covered by lines in such a way that the union of the set of lines has the same measure as the original set. This surprising and counter-intuitive result has a dual formulation in the form of a prescribed projection theorem. We investigate an analogue of these results in which lines are replaced by shifts of a fixed curve. In particular, we show that a measurable set in the plane can be covered by translations of a fixed open curve, obeying some mild curvature assumptions, in such a way that the union of the translated curves has the same measure as the original set. Our results rely on a Venetian blind construction and extend to transversal families of projections. As an application, we consider how duality between curves and points can be used to construct nonlinear Kakeya sets.

WILLIAM VERREAULT, University of Toronto
The Cesàro Operator on local Dirichlet spaces

The family of Cesàro operators σ_n^α, $n \geq 0$ and $0 \leq \alpha \leq 1$, consists of finite rank operators on Banach spaces of analytic functions on the open unit disc. We investigate these operators as they act on the local Dirichlet spaces $\mathcal{D}_\zeta$. It is well-established that they provide a linear approximation scheme when $\alpha > \frac{1}{2}$, with the threshold value $\alpha = \frac{1}{2}$ being optimal. We strengthen this result by deriving precise asymptotic values for the norm of these operators when $\alpha \leq \frac{1}{2}$, corresponding to the breakdown of approximation schemes. Additionally, we establish upper and lower estimates for the norm when $\alpha > \frac{1}{2}$.

This is joint work with Eugenio Dellepiane, Javad Mashreghi, and Mostafa Nasri.

MAHISHANKA WITHANACHCHI, University of Calgary
Vanishing Cohomology and the Corona Problem for the Algebra of Bounded Holomorphic Functions on the Polydisk

In this talk, we study the Corona problem for the Banach algebra $H^\infty(\mathbb{D}^n)$ of bounded holomorphic functions on the polydisk $\mathbb{D}^n \subset \mathbb{C}^n$. In this setting, the Corona problem asks whether the polydisk $\mathbb{D}^n$ is dense in the Gelfand topology in the maximal ideal space of $H^\infty(\mathbb{D}^n)$. We present new necessary and sufficient conditions under which the problem can be solved. An important part of our work is a new result on the vanishing of the first cohomology of a sheaf of germs of holomorphic functions on the n-fold Cartesian product of the maximal ideal space of $H^\infty(\mathbb{D})$. Our method is based on a new important result on the solution of special $\overline{\partial}$ equations on a polydisk. This is a joint work with Alex Brudnyi.

ZHICHUN ZHAI, MacEwan University

Stengthened Fractional Sobolev Inequalities and Geometric Inequalities

This study has two primary objectives. The first is to enhance fractional Sobolev-type inequalities in Besov spaces using the framework of classical Lorentz spaces. In this process, we establish that the Sobolev inequality in Besov spaces is equivalent to the fractional Hardy inequality and an iso-capacitary-type inequality.

The second objective is to strengthen fractional Sobolev-type inequalities in Besov spaces through capacitary Lorentz spaces associated with Besov capacities. To achieve this, we first analyze the embedding of the associated capacitary Lorentz space into the classical Lorentz space. Subsequently, we establish the embedding of the Besov space into the capacitary Lorentz space. Additionally, we demonstrate that these embeddings are intricately connected to iso-capacitary-type inequalities, interpreted through a newly introduced fractional $(\beta, p, q)-$perimeter. Furthermore, we provide characterizations of more general Sobolev-type inequalities in Besov spaces.

NINA ZORBOSKA, University of Manitoba

Hankel measures and Hankel type operators on weighted Dirichlet spaces

I will talk about Hankel measures and the boundedness of measure induced Hankel type operators on weighted Dirichlet spaces, extending the known results for the cases of the classical Hardy and Dirichlet spaces. The approach relies on recent results on weak products of complete Nevanlinna-Pick reproducing kernel Hilbert spaces.