Modeling, Analysis, and Computation of Variational Problems

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SOOKYUNG JOO, Old Dominion University

stability of nematic state in periodically modulated nematic phases

Nematic liquid crystals composed of bent-core molecules may exhibit periodically modulated structure. One of these phases is the twist bend nematic phase where the molecules are arranged in a heliconical structure with a nanoscale pitch. This can be characterized when the bend elastic constant is much smaller than both splay and twist elastic ones. We study the model of the twist bend nematic phase that allows the bend elastic constant to be small but in the positive range and attain its minimizer in one dimensional setting. We also characterize the parameter regime for the stability of the global and local minimizers of the nematic phase under the homeotropic boundary condition. Numerical simulations based on the constrained minimization is used to illustrate the predictions of the analysis. This is a joint work with C. Garcia-Cervera, T. Giorgi, and Z. Li.

THEODORE KOLOKOLNIKOV, Dalhousie University

BO LI, University of California, San Diego

The Legendre-Transformed Poisson-Boltzmann Electrostatics

The Poisson-Boltzmann (PB) equation for continuum electrostatics is the Euler-Lagrange equation of the PB electrostatic energy functional of electrostatic potentials. The Legendre-transformed PB (LTPB) electrostatic energy functional of all electric displacements is a convex functional dual to the PB functional. It is shown that both formulations are equivalent. A penalty model based on the LTPB electrostatics is constructed and applied to the dielectric variational solvation of charged molecules. The related numerical algorithms, computational results, and convergence analysis are presented.

XIN YANG LU, Lakehead University

Geometry of minima in co-polymer models

Energies governing the behavior of copolymers often contain a local term, plus a long range interaction. Generally speaking, the former has a coagulating effect, preferring fewer but bigger components, while the latter has a splitting effect, preferring smaller, more numerous components. Therefore, optimal configurations must arrange themselves to strike a balance between those two competing forces. In this talk, we will present some recent results in this direction.

ROSSITZA MARINOVA, Concordia University of Edmonton

Variational Approach for Computing Solitary-Wave Solutions

We present a variational approach for effectively identifying solitons within a specific mathematical framework. This method involves minimizing a functional to obtain solutions that exhibit solitonic behavior. To validate the approach, we provide numerical results that demonstrate its computational efficiency and effectiveness. Our findings show that this method achieves high accuracy in solving the relevant problems, highlighting its robustness in capturing key features of soliton dynamics. As a result, the approach proves to be both accurate and highly efficient across a range of soliton-related problems.

HANSOL PARK, Simon Fraser University

Emergent behavior of mathematical models on manifolds

In this talk, I consider a free energy functional on Cartan-Hadamard manifolds, and investigate the existence of its global minimizers. The energy functional consists of two components: an entropy (or internal energy) and an interaction energy modelled by an attractive potential. The two components have competing effects, as they favour spreading by linear diffusion and blow-up by nonlocal attractive interactions, respectively. I introduce necessary and sufficient conditions for existence of ground states for manifolds with sectional curvatures bounded above and below, respectively. In particular, for general Cartan-Hadamard manifolds, superlinear growth at infinity of the attractive potential prevents the spreading. The behaviour can be relaxed for homogeneous manifolds, for which only linear growth of the potential is sufficient for this purpose.

GUANYING PENG, Worcester Polytechnic Institute
A regularizing property of the 2D Eikonal equation

The 2D Eikonal equation is closely related to the variational analysis of a classical energy functional, namely, the Aviles-Giga functional in connection with smectic liquid crystals and thin film blisters. In the variational setting, significant effort has been devoted towards understanding solutions of the 2D Eikonal equation with low fractional Besov regularity. Notably, weak solutions under certain low regularity conditions exhibit automatic regularization. In this talk, I will present a new regularizing effect for weak solutions of the 2D Eikonal equation under a weak fractional Besov regularity. This regularity lies at the borderline between continuity and the presence of vortex singularities. This is joint work with Xavier Lamy and Andrew Lorent.

CHONG WANG, Washington and Lee University
Core Shells, Double Bubbles, and Lens Clusters in Ternary Nonlocal Isoperimetric Problems

We study a two-dimensional inhibitory ternary system characterized by a free energy functional that combines a short-range interface interaction energy, which promotes micro-domain growth, with a Coulomb-type long-range interaction energy that prevents the unlimited spreading of micro-domains. We analyze two distinct scenarios. In the first scenario, two species are vanishingly small while one species is dominant. We investigate the global minimizers of the associated ternary local isoperimetric problem and demonstrate how the geometry of minimizers evolves as surface tensions vary. This transition progresses from symmetric double bubbles, through asymmetric double bubbles, to core-shell structures. We then examine the influence of nonlocal interactions, focusing particularly on a degenerate case where minimizers exhibit a core-shell geometry, as this phase configuration aligns with physical expectations for nonlocal ternary systems. In the second scenario, two species are dominant, and one species is vanishingly small. In this case, we distinguish two energy levels: the zeroth-order energy, which encodes the optimal arrangement of the dominant constituents, and the first-order energy, which determines the shape of the vanishing constituent. We also shows that, for any optimal configuration, the vanishing phase must lie at the boundary between the two dominant constituents, forming lens clusters or vesica piscis.

ZHICHUN ZHAI, MacEwan University
A nonlinear equation induced by fractional $p-$convexity

In our study of the connection between fractional convexity and the fractional p-Laplace operator, we derive a nonlocal and nonlinear equation. We begin by proving the existence and uniqueness of the viscosity solution to this equation. Subsequently, we demonstrate that $u(x)$ is a viscosity sub-solution of the equation if and only if $u(x)$ possesses the property of $(\alpha, p)-$convexity. Finally, we characterize the viscosity solution of this equation as the envelope of an $(\alpha, p)-$convex sub-solution. Our approach leverages the attainability of the exterior data and a comparison principle for the nonlocal, nonlinear equation.