HANSOL PARK, Simon Fraser University
Emergent behavior of mathematical models on manifolds

In this talk, I consider a free energy functional on Cartan-Hadamard manifolds, and investigate the existence of its global minimizers. The energy functional consists of two components: an entropy (or internal energy) and an interaction energy modelled by an attractive potential. The two components have competing effects, as they favour spreading by linear diffusion and blow-up by nonlocal attractive interactions, respectively. I introduce necessary and sufficient conditions for existence of ground states for manifolds with sectional curvatures bounded above and below, respectively. In particular, for general Cartan-Hadamard manifolds, superlinear growth at infinity of the attractive potential prevents the spreading. The behaviour can be relaxed for homogeneous manifolds, for which only linear growth of the potential is sufficient for this purpose.