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Core Shells, Double Bubbles, and Lens Clusters in Ternary Nonlocal Isoperimetric Problems

We study a two-dimensional inhibitory ternary system characterized by a free energy functional that combines a short-range interface interaction energy, which promotes micro-domain growth, with a Coulomb-type long-range interaction energy that prevents the unlimited spreading of micro-domains. We analyze two distinct scenarios. In the first scenario, two species are vanishingly small while one species is dominant. We investigate the global minimizers of the associated ternary local isoperimetric problem and demonstrate how the geometry of minimizers evolves as surface tensions vary. This transition progresses from symmetric double bubbles, through asymmetric double bubbles, to core-shell structures. We then examine the influence of nonlocal interactions, focusing particularly on a degenerate case where minimizers exhibit a core-shell geometry, as this phase configuration aligns with physical expectations for nonlocal ternary systems. In the second scenario, two species are dominant, and one species is vanishingly small. In this case, we distinguish two energy levels: the zeroth-order energy, which encodes the optimal arrangement of the dominant constituents, and the first-order energy, which determines the shape of the vanishing constituent. We also shows that, for any optimal configuration, the vanishing phase must lie at the boundary between the two dominant constituents, forming lens clusters or vesica piscis.