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Counting lattice paths with respect to a linear boundary of rational slope

There is a remarkable and well-known enumeration result regarding lattice paths with unit up and right steps: the number of paths from $(0,0)$ to (g,g) that contain $2k$ steps above the line $y = x$ does not depend on the integer k. This result — called the Chung-Feller theorem — has inspired numerous authors to search for similar patterns throughout the years. In this accessible talk, we discuss some of our recent findings regarding paths that end at (ga, gb) and respect the boundary line given by $y = \frac{b}{a}x$ for coprime integers a, b; these findings include a result similar to the Chung-Feller theorem and an enumeration formula that generalizes counts conducted by previous authors Grossman and Bizley. In addition to discussing these results, we explain (at a high level) the combinatorial methods we used to obtain them and reveal a connection between our formula and the study of symmetric functions.

This is joint work with Jonathan Jedwab and Amarpreet Rattan.