SHAHABODDIN SHAABANI, Concordia University
The Operator Norm of Paraproducts on Bi-parameter Hardy Spaces

In this talk, we discuss the recent work on the operator norm of bi-parameter paraproducts on bi-parameter Hardy spaces. A paraproduct is a bi-linear form arising in the product of two functions both expanded in either a wavelet basis such as the Haar or in the Littlewood-Paley pieces. In the one-parameter theory, the frequency interactions in the product of two functions are divided into either low-low, low-high or high-high interactions, and each gives rise to a bi-linear form called a one parameter paraproduct. These forms behave much better than the product itself and for them the Holder's inequality extends to the full range of Hardy spaces.

In my recent work, it is shown that for $0 < p, q, r < \infty$, with $\frac{1}{q} = \frac{1}{p} + \frac{1}{r}$, the operator norm of the dyadic bi-parameter paraproduct of the form

$$\pi_g(f) := \sum_R \langle g, h_R \rangle \langle f \rangle_R h_R,$$

from the bi-parameter dyadic Hardy space $H^p_d(\mathbb{R} \otimes \mathbb{R})$ to $H^q_d(\mathbb{R} \otimes \mathbb{R})$ is comparable to $\|g\|_{H^r_d(\mathbb{R}\otimes\mathbb{R})}$. In the above, the some is taken over all dyadic rectangles in the plane, h_R is the bi-parameter Haar wavelet supported on R, and $\langle f \rangle_R$ is the average of f over R. It is also proved that for all $0 < p < \infty$, there holds

$$\|g\|_{BMO_d(\mathbb{R}\otimes\mathbb{R})} \simeq \|\pi_g\|_{H^p_d(\mathbb{R}\otimes\mathbb{R}) \to H^p_d(\mathbb{R}\otimes\mathbb{R})},$$

where $BMO_d(\mathbb{R} \otimes \mathbb{R})$, stands for the dyadic product BMO space. Similar results are obtained for bi-parameter Fourier paraproducts of the same form.