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On Odd Normal Numbers

A real number x is considered normal in an integer base $b \geq 2$ if its digit expansion in this base is "equitable", ensuring that for each $k \geq 1$, every ordered sequence of k digits from $\{0, 1, \ldots, b-1\}$ occurs in the digit expansion of x with the same limiting frequency. Borel's classical result asserts that Lebesgue-almost every $x \in [0, 1]$ is normal in every base $b \geq 2$. We consider the set $N(O, E)$ of reals that are normal in odd bases but not in even ones. It is known that this set has full Hausdorff dimension but zero Fourier dimension. The latter condition means that $N(O, E)$ cannot support a probability measure whose Fourier transform has power decay at infinity. Our main result is that $N(O, E)$ supports a Rajchman measure μ, whose Fourier transform $\widehat{\mu}(\xi)$ approaches 0 as $|\xi| \to \infty$ by definiton. Moreover, the decay rate of $\widehat{\mu}$ is essentially optimal, subject to the constraints of its support. The methods draw inspiration from the number-theoretic results of Schmidt and a construction of Lyons. As a consequence, $N(O, E)$ emerges as a set of multiplicity, in the sense of Fourier analysis. This addresses a question posed by Kahane and Salem in the special case of $N(O, E)$. This is a joint work with Professor Malabika Pramanik.