JACOB DENSON, University of Wisconsin, Madison

A Characterization of Boundedness For Multipliers of Spherical Harmonic Expansions

Any function on the sphere S^d has a unique orthogonal expansion of the form $f = \sum_{k=0}^{\infty} H_k f$, where $H_k f$ is a spherical harmonic of degree k. For a function $m : (0, \infty) \to \mathbb{C}$, define a family of operators T_R on S^d for each $R > 0$ by setting $T_R f = \sum m(k/R) H_k f$. Such operators are called 'multiplier operators for spherical harmonic expansions', or 'zonal convolution operators'. For $d \geq 4$, and a limited range of L^p spaces, we find necessary and sufficient conditions for the operators $\{T_R\}$ to be uniformly bounded on $L^p(S^d)$. As a consequence, we prove a 'transference principle', that the operators $\{T_R\}$ are uniformly bounded on $L^p(S^d)$ if and only if the radial Fourier multiplier operator on $\mathbb{R}^d$ with symbol $m(|\cdot|) : \mathbb{R}^d \to \mathbb{C}$ is bounded on $L^p(\mathbb{R}^d)$. Such necessary and sufficient conditions, and the resulting transference principle, are unknown for analogous spectral multipliers on any compact manifold and any $p \neq 2$. Our proof methods involve using the theory of Fourier integral operators to find variable-coefficient analogues of strategies for bounding radial Fourier multiplier operators on Euclidean space.