Graph Structure and Algorithms
(Org: **Richard Brewster** (Thompson Rivers University), **Benjamin Cameron** (University of Prince Edward Island) and/et **Kathie Cameron** (Wilfrid Laurier University))

IAIN BEATON, Acadia University
Reconfiguration Graphs for Minimal Domination Sets

A dominating set S in a graph is a subset of vertices such that every vertex is either in S or adjacent to a vertex in S. A minimal dominating set M is a dominating set such that $M - v$ is not a dominating set for all $v \in M$. In this talk we introduce a reconfiguration graph $\mathcal{R}(G)$ for minimal dominating sets under a generalization of the token sliding model. We give some preliminary results which include showing that $\mathcal{R}(G)$ is connected for trees and split graphs. Additionally we classify all graphs which have $\mathcal{R}(G) = K_n$ and $\mathcal{R}(G) = \overline{K_n}$ for all n.

BENJAMIN CAMERON, University of Prince Edward Island
Vertex-critical graphs in co-gem-free graphs

In this talk, we will show that there are only finitely many k-vertex-critical (co-gem, H)-free graphs for all k when H is any graph of order 4 by showing finiteness in the three remaining open cases, those are the cases when H is $2P_2$, $K_3 + P_1$, and K_4. Here a graph G is k-vertex-critical if $\chi(G) = k$ but $\chi(G - v) < k$ for all $v \in V(G)$ and (G, H)-free if it contains no induced subgraph isomorphic to G or H. The co-gem is the the disjoint union of a path of order 4 and a single vertex. For the first two cases we actually prove the stronger results:

- There are only finitely many k-vertex-critical (co-gem, paw$+P_1$)-free graphs for all k and that only finitely many k-vertex-critical (co-gem, paw$+P_1$)-free graphs for all $k \geq 1$.

- There are only finitely many k-vertex-critical (co-gem, P_5, $P_3 + cP_2$)-free graphs for all $k \geq 1$ and $c \geq 0$.

Our results imply the existence of simple polynomial-time certifying algorithms to decide the k-colourability of (co-gem, H)-free graphs for all k and all H of order 4 by searching for the vertex-critical graphs as induced subgraphs. As time allows, we will sketch some of the new ideas used in our proofs, including an application of Sperner's Theorem on the number of antichains in a partially ordered set.

KATHIE CAMERON, Wilfrid Laurier University
Frozen Colourings

A k-colouring of a graph is an assignment of at most k colours to its vertices so that the ends of each edge get different colours. We consider the question: When it is possible to obtain any k-colouring from any other by changing the colour of one vertex at a time, while always having a k-colouring? This is equivalent to asking whether the "reconfiguration graph" is connected: The reconfiguration graph of the k-colourings, denoted $R_k(G)$, is the graph whose vertices are the k-colourings of G, and two colourings are adjacent in $R_k(G)$ if they differ in colour on exactly one vertex.

A k-colouring is called frozen if there is no vertex whose colour can be changed so that the result is still a k-colouring. A frozen colouring corresponds to an isolated vertex of the reconfiguration graph. Equivalently, a frozen k-colouring is a partition of the vertices into at most k sets, each of which is both independent and dominating. The terms fall colouring and indominable have also been used. We have found several new classes of graphs with frozen colourings and an operation which transforms a k-chromatic graph with a frozen $(k + 1)$-colouring into a $(k + 1)$-chromatic graph with a frozen $(k + 2)$-colouring, without using the "join" operation or adding universal vertices. I will discuss the recently resolved problem: For which values of k and t does there exist a k-colourable graph with no induced path on t vertices with a frozen $(k + 1)$-colouring?

This is joint work with Manoj Belavadi and Elias Hildred.

NANCY CLARKE, Acadia University
On the Structure of Dominating Graphs of Trees and Cycles

The dominating graph of a graph G has as its vertices all dominating sets of G, with two vertices adjacent if the corresponding dominating sets differ by the addition or deletion of a single vertex of G. We are interested in the properties of such graphs. In particular, we show that the dominating graph of any tree has a Hamilton path and that the dominating graph of a cycle on n vertices has a Hamilton path if and only if n is not a multiple of 4. Joint work with K. Adaricheva, H. Smith Blake, C. Bozeman, R. Haas, M. Messinger, and K. Seyffarth.

CÉSAR HERNÁNDEZ CRUZ, Universidad Nacional Autónoma de México
Full homomorphisms to trees

Given two graphs G and H, a full homomorphism from G to H is a function $\varphi\colon V_G \to V_H$ which preserves adjacencies and non-adjacencies. When a full homomorphism exists from G to H we say that G is fully H-colourable; the full H-colouring problem is the problem of deciding whether an input graph G admits a full H-colouring for a fixed graph H. In 2008, Feder and Hell proved that there are always finitely many minimal obstructions to the full H-colourability problem, and moreover, the order of any such minimal obstruction is at most $|V_H| + 1$. Thus, there is a brute force polynomial time algorithm to solve the full H-colouring problem.

In this talk we consider the full H-colouring problem when H is a tree; in particular, we are interested in the optimal time for recognizing a fully H-colourable graph. We present a linear time algorithm to solve the full H-colouring problem when H is a path. We also define the class of *fully tree-colourable graphs* as the family of graphs admitting a full homomorphism to some tree, and exhibit a characterization in terms of minimal induced subgraphs for such a class.

ANGÈLE FOLEY, Wilfrid Laurier University
When is a graph e-positive?

In 1995 Stanley introduced the chromatic symmetric function of a graph and posed a central question: when can it be written as a linear combination of elementary symmetric functions using only positive coefficients? When it can, the graph is called e-positive. He also presented the Stanley-Stembridge conjecture that a particular kind of clawfree graph is e-positive. In 2024 Hikita announced a proof of this conjecture, but along the way there have been many related results. This talk will explore our 2019 contribution with Hoang and Merkel, as well as some more recent developments.

PAVOL HELL, Simon Fraser University
Signed Graphs and Homomorphisms

Signed graphs first arose in the theory of social balance, and also arise in clustering in networks, root systems, matroids, and flows on non-orientable surfaces. They offer a refined view of many basic graph theory results. They are particularly intriguing from the perspective of graph homomorphisms. I will focus on the complexity dichotomy problem, discuss the solution of the basic homomorphism problem, and the new results and remaining challenges for the list homomorphism problem. The new results are joint with Jan Bok, Rick Brewster, and Nikola Jedličková.

KIARA MCDONALD, University of Victoria
Broadcast Independence in Different Classes of Graphs

In the area of Graph Theory, the well-known problems of domination, packing and independence are generalized by broadcast domination, broadcast packing and broadcast independence. As an analogy and application, consider a city, where one wants to place cell towers of different signal strengths subject to certain conditions. If every building in the city hears the signal from at least (respectively at most) one tower, then the broadcast is dominating (respectively packing). If no tower hears the signal

from another tower, the broadcast is independent. The sum of the tower signal strengths is called the cost of the broadcast. The total cost of a maximum independent broadcast is called the broadcast independence number.

Our research was focused on determining explicit formulas and polynomial time algorithms for the broadcast independence number of various types of graphs. This parameter is difficult to compute for graphs in general, so we restrict the problem to specific classes of graphs to make use of their special structural properties to solve the problem. For a graph from a given class, we constructed a new graph, called the broadcast ball intersection graph. We were able to show that if the broadcast ball intersection graph is weakly chordal, then broadcast independence is polynomial time solvable for the given class of graphs. This talk is based on joint work with Richard Brewster (TRU) and Jing Huang (UVic).

BEN MOORE, Institute of Science and Technology Austria
Orientations of Highly Edge Connected Graphs

A nowhere zero 3 flow (henceforth: NZ3F) is an orientation of a graph such that, at each vertex, the indegree minus the outdegree is divisible by 3. Grotzsch's Theorem says that every triangle-free planar graph is 3-colourable. Tutte conjectured a wide-sweeping generalization: every 4-edge-connected graph admits a NZ3F. Lovasz, Thomassen, Wu and Zhang proved that 6-edge-connected graphs admit such a flow. We extend this result by showing that one can allow arbitrarily many 5-edge-cuts or 4-edge-cuts — under some technical conditions.

Our theorem generalizes the more technical version of the Lovasz, Thomassen, Wu, Zhang theorem.

This is joint work with Soffia Arnadottir, Evelyne Smith Roberge, Zdenek Dvorak, Bernard Lidicky, and Robert Samal.

KATHRYN NURSE, Simon Fraser University
Nowhere-zero flows and group connectivity - an intermediate step

We give an overview of nowhere-zero flows and group-connectivity. We discuss a generalization of Seymour's 6-flow theorem. This generalization is an intermediate step between group connectivity and nowhere-zero flows, where the user may prescribe certain boundary constraints on the vertices. Based on joint work with M. DeVos (Simon Fraser University), and J. McDonald (Auburn University).

SHANNON OGDEN, University of Victoria
The Rainbow Connection

Given a graph H, an edge-coloured graph G is H-rainbow saturated if it does not contain a rainbow copy of H, but the addition of any non-edge in any colour creates a rainbow copy of H. The rainbow saturation number, denoted by rsat(n, H), is the minimum number of edges among all H-rainbow saturated edge-coloured graphs on n vertices. We will prove that, for any non-empty graph H, the rainbow saturation number is linear in n. This confirms a recent conjecture of Girão, Lewis, and Popielarz. Based on joint work with Natalie Behague, Tom Johnston, Shoham Letzter, and Natasha Morrison.

BEN SEAMONE, Dawson College
Ramsey numbers of signed graphs

A signed graph is a pair (G, σ) where $G = (V, E)$ is a graph and $\sigma : E(G) \to \{+, -\}$ is a signature which assigns a sign to each edge of G. One well-studied operation on signed graphs is that of switching at a vertex $v \in V(G)$, by which we mean that every edge incident to v has its sign changed. Two signed graphs are called equivalent if one can be obtained from the other by a sequence of vertex switches.

We call a complete signed graph positive (negative) if every edge is positive (negative). We study the following Ramsey problem on signed graphs – for positive integers s and t, what is the smallest n such that every signed complete graph on n vertices has an equivalent signed complete graph containing either a negative K_s or positive K_t. This "signed Ramsey number" is denoted $r_\pm(s, t)$. We show how a variety of approaches lead to upper and lower bounds on $r_\pm(s, t)$, settle an open problem by establishing the exact value of $r_\pm(4, t)$ for every t, and determine the asymptotics of $r_\pm(5, t)$ and $r_\pm(6, t)$.