## Geometric quantization for young people

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**REEBHU BHATTACHARYYA**, University of Michigan, Ann Arbor

*Isotropic States on Kähler Manifolds*

We will define classes of (semi-classical) wave packets (whose wavefront set is a single point) on pre-quantized compact Kähler manifolds and describe a symbol calculus for them. This is a generalization of squeezed/coherent states, which have been studied extensively, especially in Euclidean space. We will generalize these further to isotropic states (whose wavefront set is an isotropic submanifold) which we define as a superposition of such wave packets, and then obtain a symbol calculus for them. Finally, we will describe our results on the overlap and Hermitian propagation of these states.

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**MICHAEL FRANCIS**, MacEwan University

*Towards $b^k$-analogues of Berezin-Toeplitz quantization*

A $b$-manifold is a smooth manifold with a specified hypersurface; its $b$-tangent bundle (Melrose's terminology) has for sections those vector fields that are tangent to the hypersurface. Scott introduced higher-order generalizations of $b$-manifolds called $b^k$-manifolds. A slightly different approach to $b^k$-manifolds that allows for global holonomy phenomena was introduced by Francis and (independently) Bischoff-del Pino-Witte. Various classical geometries extend to these settings: $b^k$-manifolds can be complex, symplectic, Riemannian etc. Geometric quantization of symplectic $b^k$-manifolds has been the subject of recent work of Guillemin-Miranda-Weitsman, Braverman-Loizides-Song and others. For complex $b^k$-manifolds, one expects analogues of Berezin-Toeplitz quantization. Systematic study of complex $b$-manifolds was initiated by Mendoza. The Newlander-Nirenberg theorem for complex $b$-manifolds was obtained by Barron-Francis and extended to the $b^k$ case by Francis. In this talk, we will survey what is known about complex $b^k$-manifolds and consider function spaces analogous to those relevant in Berezin-Toeplitz quantization.

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**ALEX KAZACHEK**, University of Waterloo

*Quantum Channel Capacities and Additivity Conjectures*

Quantum channels are a model for communicating information via the transmission of quantum states. The propensity for such a channel to send information is known as its capacity, taking several different forms depending on operational constraints. Unlike classical channels, computing such capacities is both computationally and mathematically intractable due to a phenomenon known as non-additivity. As a result, taming and bounding their capacities involves the interplay of sophisticated mathematics such as asymptotic geometric analysis, creating quantum analogues of information-theoretic notions, as well as heuristic and numerical work with low-dimensional counterexamples. I will review the history of the field and its central concepts, as well as present the modern techniques for tackling additivity problems such as log-singularity arguments and degradability. Moreover, I will briefly discuss how channels in their abstract may be connected to prior work of mine with T. Barron on coherent states in Kähler quantization.

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**HYUNMOON KIM**, University of Toronto

*Stratification of families of representations of the Heisenberg Lie algebra*

We discuss a parametrization of a family of irreducible representations of the Heisenberg Lie algebra by Poincare-Birkhoff-Witt isomorphisms, or equivalently, complex Lagrangian splittings. Complex conjugation stratifies this family, and in two dimensions, this stratification assembles well known families of representations of the canonical commutation relations. We discuss various properties of the stratification on the complex Lagrangian Grassmannian induced by complex conjugation–incidence relations, homotopy type of each stratum, and discrete symmetries between preferred basepoints. We speculate the potential role of the family as a classifying space for quantizations.

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**MATTHEW KOBAN**, University of Toronto
*Bundle representations of double quivers*

Representations of finite quivers in the category of vector spaces have been used to great effect in mathematical physics, geometric representation theory, and beyond. In this talk I will discuss a symplectic construction of moduli spaces of representations of quivers in the category of vector bundles over a Riemann surface. For specific quivers these spaces recover both moduli spaces of flat connections, as well as moduli spaces of Higgs bundles, each of which has a rich interaction with geometric quantization. At the same time quiver varieties themselves (in the ordinary sense) can be quantized and one can ask about the extent to which this more general construction admits any reasonable quantization. This is work in progress joint with Lisa Jeffrey and Steven Rayan.

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**ETHAN ROSS**, University of Toronto
*Quantization of Symplectic Stratified Spaces*

Symplectic stratified spaces are a natural class of singular spaces which appear in equivariant symplectic geometry. There have been many approaches to trying to quantize stratified symplectic spaces that can be quite ad-hoc and rely on a quotient structure. In this talk I will discuss a new more intrinsic way of quantizing that generalizes the more classical approaches for manifolds and doesn't require a group action, but still functions nicely in the presence of one.

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**KALEB D. RUSCITTI**, University of Waterloo
*Degeneration of Holomorphic Sections to Bohr-Sommerfeld points for Moduli of SL(2,C) Bundles*

The moduli spaces of $\mathrm{SL}(2,\mathbb{C})$ bundles on a compact Riemann surface ($g > 1$) are one example of the invariance of polarization principle: Jeffrey and Weitsman showed the number of Bohr-Sommerfeld points is equal to the Verlinde formula, which counts the dimension of the Kähler quantization. This is a numerical equivalence, but there is no canonical isomorphism taking holomorphic sections to Bohr-Sommerfeld points. Their proof uses a moment polytope coming from the Goldman flows associated to a trinion decomposition of the Riemann surface, but existing proofs of the Verlinde formula are not clearly related to this polytope.

Biswas and Hurtubise recently showed that by degenerating the Riemann surface along a trinion decomposition, one obtains a degeneration of the moduli space to a toric variety of framed parabolic bundles. In this talk, we discuss how to use this degeneration at the level of holomorphic sections to degenerate a section over the moduli space into a section over the toric variety, which is directly related to the Bohr-Sommerfeld points via standard toric-geometric results. This also provides another proof of the Verlinde formula, via the moment polytope defined by the Goldman flows.

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**OOD SHABTAI**, University of Toronto
*Pairs of spectral projections of quantum observables on Riemann surfaces*

We discuss the semiclassical behaviour of pairs of spectral projections corresponding to incompatible quantum observables, in the framework of geometric quantization of closed Riemann surfaces.

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**ZHONGKAI TAO**, UC Berkeley
*Spectral asymptotics for kinetic Brownian motion*

I will talk about the kinetic Brownian motion operator which serves as an interpolation of geodesic flow and the Laplace operator on a compact Riemannian manifold. Then I will motivate open problems on how to define it on differential forms.

**DAN WANG**, IST, University of Lisbon
*Geometric Quantization on Toric Varieties*

Geometric quantization on symplectic manifolds plays an important role in representation theory and mathematical physics, deeply relating to symplectic geometry and differential geometry. A crucial problem is to understand the relationship among geometric quantizations associated with different polarizations. In this talk, we will discuss the quantum spaces associated with mixed polarizations and the large limit of quantum spaces on toric varieties.

**YU-TUNG (TONY) YAU**, University of Michigan
*Berezin-Toeplitz quantization in real polarizations*

A fundamental theorem in Berezin-Toeplitz quantization states that, on a compact Kaehler manifold, a unique deformation quantization is determined by its asymptotic action via Toeplitz operators on the quantum Hilbert space in Kaehler polarization. Since Schlichenmaier proved this result, numerous research in this area has been emerging. In this talk, I will discuss how to construct Toeplitz type operators beyond the case of Kaehler polarizations so as to obtain an analogue to Schlichenmaier's result. I will especially focus on compact symplectic manifolds with a pair of transversal real polarizations. Noting that the construction of Toeplitz operators in the Kaehler case involves the inner product on the space of $L^2$ sections of the prequantum line bundle, I will also explain how to overcome the difficulty that the quantum Hilbert spaces in real polarizations are distributional sections.