JOZSEF SOLYMOSI, University of British Columbia

A sparse removal lemma for pentagons

Counting pentagons in graphs and hypergraphs is a classical problem in extremal graph theory. The following basic question has equivalent formulations using graphs or 3-uniform linear hypergraphs. Here, we use the hypergraph notation to state our main result. A 3-uniform hypergraph is linear if any two edges have at most one vertex common. A C_5 in a triple system is a set of ten distinct vertices $\{u_1, \dots, u_5, v_1, \dots, v_5\}$ and five (distinct) edges $u_i u_{i+1} v_i$ where $i \in [5]$ and indices are taken modulo 5. We say that this C_5 is an expansion of $u_1 \dots u_5$. Given a hypergraph H, write $e(H)$ and $d(H)$ for the number of edges and average degree of H, respectively.

Let $n > 10$ and let H be an n-vertex linear triple system with $m > 100\, n^{3/2}$ edges. Then the number of copies of C_5 in H is at least m^6/n^7.

This result has interesting applications in additive combinatorics and discrete geometry. Joint work with Dhruv Mubayi.