Geometric Analysis and PDE

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AILANA FRASER, University of British Columbia

Minimal surfaces in higher codimension

I will describe recent progress on the stability theory for minimal surfaces in higher codimension, in particular on Bernstein theorems, and applications of minimal surfaces in higher codimension in Riemannian geometry.

STEPHEN GUSTAFSON, University of British Columbia

Two-solitons with logarithmic separation for 1D NLS with repulsive delta potential

For the nonlinear Schrodinger equation in one dimension, with a repulsive delta potential that is not too strong, we show the existence of two-soliton solutions with logarithmic (in time) separation. The construction is based on that of Nguyen for the case without potential, modified to account for the additional interaction between the potential and the solitons. This interaction manifests through a perturbed translational eigenfunction, whose detailed properties play a key role. This is joint work with Takahisa Inui.

SLIM IBRAHIM, University of Victoria

Persistence of vorticity concentration in the two-point vortex system of the 2D Euler equations

In this talk, I will investigate the vortex confinement property of solutions to the 2D Euler equations, specifically focusing on the stability around the critical point. For the two-point vortex system, I will analyze the duration over which vorticity concentration is sustained and show that, regardless of the vorticity strength, this concentration persists indefinitely. This is a joint work with R. Qin and S. Shen.

KENNEDY IDU, University of Toronto

Optimality results on Alexandrov's estimate

The classical Alexandrov's estimate states that if Ω is a bounded open convex domain in $\mathbb{R}^n$, and $u : \bar{\Omega} \to \mathbb{R}$ is a convex function vanishing on the boundary $\partial\Omega$, then

$$[u]_{1/n}^n \leq C(\Omega)|\partial u(\Omega)|.$$

Here $C(\Omega)$ is a constant depending only on Ω, ∂u denotes the subgradient of u and

$$[u]_\alpha := \sup_{x,y\in\Omega, x\neq y} \frac{|u(x)-u(y)|}{|x-y|^\alpha}. \tag{1}$$

The estimate is not only crucial to regularity theory of the Monge-Ampere equation, but also main tool for some linear elliptic PDE estimates. In this talk, we discuss some recent optimality and sharpness results from extension and refinements of the estimate depending on the geometry of $\partial\Omega$. This is a joint work with Charles Griffin and Robert L. Jerrard (University of Toronto).

ILMARI KANGASNIEMI, University of Cincinnati

On the theory of quasiregular values

A quasiregular (QR) map is a Sobolev map f between domains of $\mathbb{R}^n$ satisfying the distortion inequality $|Df(x)|^n \leq K\det(Df(x))$ at almost every x, where $K \geq 1$ is a constant. QR maps form a higher-dimensional class of maps with many similar geometric properties as single-variable holomorphic maps. In this talk, we consider a generalization of the distortion inequality of the form $|Df(x)|^n \leq K\det(Df(x)) + \Sigma(x)|f(x)-y|^n$, where Σ is a real-valued weight function and $y \in \mathbb{R}^n$ is a fixed point. Our recent results show that under various L^p-integrability assumptions on Σ, this condition can be used to prove single-value counterparts to many fundamental results of QR-maps at the point y. The list of generalized results includes e.g. the QR-versions of the open mapping theorem, Liouville theorem, Picard theorem, and the small K -theorem. Joint work with Jani Onninen.

CHENKUAN LI, Brandon University
The analytical solution to the multi-term time-fractional diffusion-wave equation

Applying the inverse operator method and the multivariate Mittag-Leffler function, we derive the analytic solution for the following multi-term time-fractional diffusion-wave equation in the Caputo fractional derivative sense:

$$\begin{cases} {}_c\dfrac{\partial^\rho}{\partial t^\rho}M(t,\sigma) + \displaystyle\sum_{j=1}^{m} \lambda_j \, {}_c\dfrac{\partial^{\rho_j}}{\partial t^{\rho_j}}M(t,\sigma) = \triangle M(t,\sigma) + g(t,\sigma), \\ M(0,\sigma) = \theta(\sigma), \quad M_t'(0,\sigma) = \beta(\sigma), \end{cases}$$

where $\triangle = \displaystyle\sum_{i=1}^{n} \frac{\partial^2}{\partial \sigma_i^2}$, all λ_j for $j = 1, 2, \cdots, m$ are arbitrary constants, and $1 < \rho_1 < \rho_2 < \cdots < \rho_m < \rho \leq 2$. In particular if $\lambda_1 = \cdots = \lambda_m = 0$ and $\rho = 2$, then the above equation turns out to be the non-homogeneous wave equation in $\mathbb{R}^n$:

$$\begin{cases} \dfrac{\partial^2}{\partial t^2}M(t,\sigma) = \triangle M(t,\sigma) + g(t,\sigma), \\ M(0,\sigma) = \theta(\sigma), \quad M_t'(0,\sigma) = \beta(\sigma), \end{cases}$$

which has the uniform solution for all $n \geq 1$

$$M(t,\sigma) = \sum_{s=0}^{\infty} I_t^{2s+2} \triangle^s g(t,\sigma) + \sum_{s=0}^{\infty} \frac{t^{2s}}{(2s)!} \triangle^s \theta(\sigma) + \sum_{s=0}^{\infty} \frac{t^{2s+1}}{(2s+1)!} \triangle^s \beta(\sigma),$$

where I_t^{2s+2} is the Riemann–Liouville fractional partial integral operator. We further show that the solution given above coincides with the classical results, such as d'Alembert and Kirchoff's formulas, with an example demonstrating its power and simplicity.

CALEB MARSHALL, University of British Columbia
The Size of Spanning Sets of Lines for Fractal Subsets of $\mathbb{R}^n$

Given $X \subset \mathbb{R}^n$, we let $L(X)$ denote the family of lines in $\mathbb{R}^n$ which contain at least two points of X. We call this line family $L(X)$ the *spanning set of lines* for X. In some sense, $L(X)$ is the "greediest possible" cover of X by lines; and so, it makes sense to ask: how large (relative to the size of X) must this set $L(X)$ be?

If X has finite cardinality, a result from the mid 1980's known as Beck's Theorem states: either X is essentially contained in a single line, or else $L(X)$ has cardinality $\sim |X|^2$. Recent works of T. Orponen, P. Shmerkin, and H. Wang, as well as K. Ren, establish a continuum Beck Theorem for fractal sets X, bounding the Hausdorff dimension of $L(X)$ in terms of the dimension of X.

In this talk, we discuss our recent improvement of these continuum Beck Theorems–improvements which are conditional upon our fractal sets $X \subset \mathbb{R}^n$ satisfying more restrictive anti-concentration conditions on hyperplanes. We also construct examples showing our estimate can be sharp for all possible values of Hausdorff dimension.

We conclude the talk by discussing a new conjectured *equality* for the Hausdorff dimension of $L(X)$. This conjecture is compatible with the previous results of Orponen, Shmerkin and Wang, Ren, and Bright-M, and suggests a decomposition of fractal sets into concentrated and anti-concetrated parts.

This talk is based on joint work with P. Bright (UBC).

YUVESHEN MOOROOGEN, University of British Columbia
A large-scale variant of the Erdos similarity conjecture

Consider a sequence of real numbers increasing to infinity. How large can a subset of the real line be before it is forced to contain some affine image of that sequence? This question fits into a huge body of work in analysis and number theory concerned with constructing large sets that fail to contain prescribed structures. I will discuss recent progress on this question and comment on its connections with a now 50-year old open problem of Erdos.

SERGII MYROSHNYCHENKO, University of the Fraser Valley
Centroid of a convex body can be rarely the centroid of its sections

We construct a convex body K in $\mathbb{R}^n$, $n \geq 5$, with the property that there is exactly one hyperplane H passing through $c(K)$, the centroid of K, such that the centroid of $K \cap H$ coincides with $c(K)$. This provides answers to questions of Grunbaum and Loewner for $n \geq 5$. The proof is based on the existence of non-intersection bodies in these dimensions. Joint work with K. Tatarko and V. Yaskin.

JOSE PALACIOS, University of Toronto
Linearized dynamic stability for vortices of Ginzburg-Landau evolutions

We consider the problem of dynamical stability for the vortex of the Ginzburg-Landau model. Vortices are one of the main examples of topological solitons, and their dynamic stability is the basic assumption of the asymptotic "particle plus field" description of interacting vortices. In this talk we focus on co-rotational perturbations of vortices and establish a variety of pointwise dispersive and decay estimates for their linearized evolution in the relativistic (or Klein-Gordon) case. One of the main ingredients is the construction of the distorted Fourier transform associated with the (two) linearized operators at the vortex. The general approach follows that of Krieger-Schlag-Tataru and Krieger-Miao-Schlag in the context of stability of blow-up for wave maps and relies on the spectral analysis of Schrodinger operators with strongly singular potentials (see also Gezstesy-Zinchenko). However, since the vortex is not given by an explicit formula, and one of the operators appearing in the linearization has zero energy solutions that oscillate at infinity, the linear analysis requires some additional work. In particular, to construct the distorted Fourier basis and to control the spectral measure some additional arguments are needed, compared to previous works. This is joint work with Fabio Pusateri.

CRISTIAN RIOS, University of Calgary
Continuity of solutions to infinite degenerate elliptic equations in the plane

We obtain the continuity of weak solutions to infinite degenerate quasilinear equations

$$-\mathrm{div}\mathcal{A}\left(x,u\right)\nabla u=\phi_0-\mathrm{div}_A\vec{\phi}_1$$

where one of the eigenvalues of the elliptic matrix $\mathcal{A}$ is allowed to vanish to infinite order as x approaches the vertical axis. This is an application of an abstract result obtained in all dimensions $n \geq 2$. The Carnot-Carathéodory metric associated with the operator is highly non-doubling, so traditional methods have to be adapted to Orlicz-Sobolev embeddings with gains smaller than any power $p > 1$. In particular, our methods include the first realization of a Moser iteration technique in such infinite degenerate geometries. This work was done in collaboration with Lyudmila Korobenko, Eric Sawyer, and Ruipeng Shen.

SCOTT RODNEY, Cape Breton University
Existence, Boundedness, and Regularity - an overview of some recent results in Partial Differential Equations

In this talk I will discuss joint work with D. Cruz-Uribe (Alabama), Y. Zeren, S. Cetin, F. Dal (Yildiz Technical Institute). This work surrounds existence and regularity of weak solutions to linear degenerate elliptic PDEs of the form

$$-v^{-1}\mathrm{Div}(Q(x)\nabla u) + K(x,u,\nabla u) = F$$

in a bounded domain Ω where Q is a symmetric non-negative definite measurable matrix of coefficient functions, v is a weight on Ω, K defines 1^{st} and 0^{th}-order terms, and where the data function F may take different forms. I will put this work in contrast with recent results in the area.

SIDDHARTH SABHARWAL, Texas A&M University
Existence and Asymptotics of Nonlinear Helmholtz Eigenfunctions

We discuss the problem of proving existence and asymptotics of solutions to equation $-\Delta_M u = N(u)$, where $N(u)$ is a monomial. We consider the space to be even asymptotically hyperbolic. I will introduce the main technique, which is module regularity, and how it is used for proving existence of solution, and that these nonlinear eigenfunctions have the same asymptotics as the linear eigenfunctions.

IGNACIO URIARTE-TUERO, University of Toronto
Muckenhoupt Ap weights, BMO, distance functions and related problems

I will present a new characterization of BMO (similar to existing ones) and applications we derive from it, such as solving a problem on Muckenhoupt Ap weights and distance functions. Joint work with Marcus Pasquariello.

BEATRICE-HELEN VRITSIOU, University of Alberta
On a Blaschke-Santaló-type inequality for projections of (non-symmetric) convex bodies, and some applications

Consider a convex body K in $\mathbb{R}^n$ and a projection $P_F(K)$ of it to a subspace F of $\mathbb{R}^n$. If K is not origin-symmetric, then even if we 'centre' it well (that is, 'force' K to have barycentre or Santaló point at the origin), its projection may still fail to have any of these nice properties. Therefore the classical Blaschke-Santaló inequality (which upper-bounds the product of volumes of $P_F(K)$ and of its polar set within the subspace F) cannot apply directly. We will show how to derive an essentially optimal Blaschke-Santaló inequality in such settings.

This also has applications to the existence of *regular* covering ellipsoids for not-necessarily-symmetric convex bodies (a concept introduced in a celebrated theorem of Pisier, which was proved in the symmetric case).

JOSHUA ZAHL, UBC
A survey of the Kakeya problem

A Kakeya set is a compact subset of $\mathbb{R}^n$ that contains a unit line segment pointing in every direction. The Kakeya conjecture asserts that every Kakeya set in $\mathbb{R}^n$ must have Minkowski and Hausdorff dimension n. I will survey historical and more recent results on this conjecture.