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Optimality results on Alexandrov's estimate

The classical Alexandrov's estimate states that if Ω is a bounded open convex domain in \mathbb{R}^n , and $u: \overline{\Omega} \to \mathbb{R}$ is a convex function vanishing on the boundary $\partial\Omega$, then

$$[u]_{1/n}^n \le C(\Omega) |\partial u(\Omega)|.$$

Here $C(\Omega)$ is a constant depending only on Ω , ∂u denotes the subgradient of u and

$$[u]_{\alpha} := \sup_{x,y \in \Omega, x \neq y} \frac{|u(x) - u(y)|}{|x - y|^{\alpha}}.$$
(1)

The estimate is not only crucial to regularity theory of the Monge-Ampere equation, but also main tool for some linear elliptic PDE estimates. In this talk, we discuss some recent optimality and sharpness results from extension and refinements of the estimate depending on the geometry of $\partial\Omega$. This is a joint work with Charles Griffin and Robert L. Jerrard (University of Toronto).