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The Size of Spanning Sets of Lines for Fractal Subsets of $\mathbb{R}^n$

Given $X \subset \mathbb{R}^n$, we let $L(X)$ denote the family of lines in $\mathbb{R}^n$ which contain at least two points of X. We call this line family $L(X)$ the *spanning set of lines* for X. In some sense, $L(X)$ is the "greediest possible" cover of X by lines; and so, it makes sense to ask: how large (relative to the size of X) must this set $L(X)$ be?

If X has finite cardinality, a result from the mid 1980's known as Beck's Theorem states: either X is essentially contained in a single line, or else $L(X)$ has cardinality $\sim |X|^2$. Recent works of T. Orponen, P. Shmerkin, and H. Wang, as well as K. Ren, establish a continuum Beck Theorem for fractal sets X, bounding the Hausdorff dimension of $L(X)$ in terms of the dimension of X.

In this talk, we discuss our recent improvement of these continuum Beck Theorems–improvements which are conditional upon our fractal sets $X \subset \mathbb{R}^n$ satisfying more restrictive anti-concentration conditions on hyperplanes. We also construct examples showing our estimate can be sharp for all possible values of Hausdorff dimension.

We conclude the talk by discussing a new conjectured *equality* for the Hausdorff dimension of $L(X)$. This conjecture is compatible with the previous results of Orponen, Shmerkin and Wang, Ren, and Bright-M, and suggests a decomposition of fractal sets into concentrated and anti-concetrated parts.

This talk is based on joint work with P. Bright (UBC).