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On the Thue-Vinogradov Lemma

Thue's Lemma is a helpful tool in elementary number theory. The most famous application of the lemma is to prove Fermat's theorem on sums of two squares. Vinogradov extended this Lemma to an asymmetric form. He used it in the paper "*On a general theorem concerning the distribution of the residues and non-residues of powers*", where he gave an elementary proof of the Pólya-Vinogradov inequality. Vinogradov's formulation is the following: Let p be a prime. For any $a \in \mathbb{N}$, $p \nmid a$, and $\alpha \in \mathbb{F}_p^*$, there are x, y where $x \in \{1, 2, \dots, \alpha\}$, $y \in \{1, 2, \dots, \lfloor \frac{p}{\alpha} \rfloor\}$ such that $ax \equiv \pm y \pmod{p}$.

The proof is based on a clever application of the pigeon-hole principle. We will extend this result to smaller sets and show some applications of the improved result. We will use Rédei polynomials and a simple variant of Stepanov's method for the proof.