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Symmetry-Breaking in Compartmental-Reaction Diffusion Systems with Comparable Diffusivities

Since Alan Turing's pioneering publication on morphogenetic pattern formation obtained with reaction-diffusion (RD) systems, it has been the prevailing belief that two-component reaction diffusion systems have to include a fast diffusing inhibiting component (inhibitor) and a much slower diffusing activating component (activator) in order to break symmetry from a uniform steady-state. This time-scale separation is often unbiological for cell signal transduction pathways. We modify the traditional RD paradigm by considering nonlinear reaction kinetics only inside compartments (cells) with reactive boundary conditions to the extra-compartmental space which diffusively couples the compartments via two (chemical) species. The construction of a nonlinear algebraic system for all existing steady-states, or quasi-steady-states, enables us to derive a globally coupled matrix eigenvalue problem for the growth rates of eigenperturbations from the symmetric steady-state in 1-D, 2-D, and 3-D. We show that the membrane reaction rate ratio of inhibitor rate to activator rate is a key bifurcation parameter leading to robust symmetry-breaking of the compartments. Illustrated with Gierer-Meinhardt, FitzHugh-Nagumo and Rauch-Millonas intra-compartmental kinetics, our compartmental-reaction diffusion system does not require diffusion of inhibitor and activator on vastly different time scales. Our results reveal a possible simple mechanism of the ubiquitous biological steady and oscillatory cell specialization observed in nature. (This is joint work with Michael J. Ward.)