Emerging Frontiers in Number Theory: Insights from Early-Career Researchers
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KRISTAPS BALODIS, University of Calgary
L-functions, representation theory, and geometry.

The local Langlands program predicts that one can associate representations π of p-adic groups to group homomorphisms called "L-parameters" ϕ. Moreover, one can attach complex meromorphic functions $L(\pi, s)$ and $L(\phi, s)$ to these objects, in such a way that if π is "associated" with ϕ, then $L(\pi, s) = L(\phi, s)$. It is predicted that algebraic properties of π and ϕ are encoded in the analytic behavior of the functions $L(\pi, s)$ and $L(\phi, s)$. In this talk, we will discuss recent progress on some of these conjectures which relies on a certain "geometrization" of these ideas; namely the p-adic Kazhdan-Lusztig hypothesis.

NIC BANKS, University of Waterloo
Galois Theory and Computation of Intersective Polynomials

A polynomial with integer coefficients is called intersective if it has a root modulo n for all positive integers n. This talk will focus on intersective polynomials without integer roots, which represent a local-global failure. We will discuss classification efforts for such polynomials of low degree, utilizing a Galois-theoretic characterization of Berend and Bilu (1996). We conclude by discussing data collection and computational aspects of this problem, facilitated by Sage and GAP.

DAN BARAKE, McMaster University
Characters in p-adic Vertex Operator Algebras

Vertex operator algebras (VOAs) play a central role in two-dimensional conformal field theory, however their number-theoretical properties have also garnered significant attention over the past two decades. In particular, a celebrated result of Zhu shows that the character map (i.e. 1-point correlation function or graded trace) on VOAs gives a surjection on the space of modular forms. In this talk, we will first give an introduction to these structures as well as their p-adic variants which were constructed recently by Franc and Mason. Then, we will discuss new results on the connections to p-adic modular forms, with the goal of establishing a Zhu-type theorem in this p-adic case.

JOSE CRUZ AND FATEMEH JALALVAND, University of Calgary
Geometric Properties of Log Unit Lattices

The log unit lattice is the image of the unit group of the ring of integers under the Minkowski embedding in the euclidean space. By studying Minkowski's lattice constructions, one can explore how the intrinsic algebraic features of number fields are reflected in the geometric invariants of the corresponding lattices.This perspective offers valuable insights into the arithmetic structure of number fields and has been fundamental in deriving key results, such as the celebrated class number formula.

In this talk, we will introduce parametrizing spaces for certain families of number fields, within which the associated log unit lattices live. This framework allows us to investigate geometric properties such as orthogonality and well-roundedness.

JAMES CUMBERBATCH, Purdue University

REBECCA DELAND, University of Colorado, Boulder
Limiting Density of Elliptic Divisibility Sequences

Let $E/\mathbb{Q}$ be an elliptic curve and P be a rational point of infinite order. If we write the points $[n]P = \left(\frac{A_n}{D_n^2}, \frac{B_n}{D_n^3}\right)$, the D_n's form an elliptic divisibility sequence. In this talk, we will explore the residue classes of elliptic divisibility sequences modulo p^λ for $\lambda \geq 1$. We will then discuss how we can use elliptic curves over local fields to gain information about the residue classes as $\lambda \to \infty$.

ZHENCHAO GE, University of Waterloo
A discrete mean value for Dirichlet L-function over local extrema

The classical second integral moment of $\zeta(s)$ shows that the integral average of $|\zeta(\frac{1}{2}+it)|^2$ is $\log t$. Assuming the Riemann Hypothesis and letting γ, γ^+ be the imaginary parts of consecutive critical zeros of $\zeta(s)$, Conrey and Ghosh proved that the mean value of $|\zeta(\frac{1}{2}+it)|^2$ over the maxima between γ, γ^+ up to T is asymptotic to $\frac{1}{2}(e^2-5)\frac{T}{2\pi}\log(\frac{T}{2\pi})^2$. In other words, the discrete mean of $|\zeta(\frac{1}{2}+it)|^2$ at a critical point is $\frac{1}{2}(e^2-5)\log t$, which is a constant factor larger.

In this talk, we will demonstrate that the analogous phenomenon does not exist for the Z-function associated to a Dirichlet L-functions. Specifically, we show that the discrete mean value of Hardy's Z-function over its local extrema has an asymptotic formula with a negative leading coefficient. In contrast, Korolev and Jutila have proven that the integral mean value of Hardy's Z-function does not exhibit such behavior. Moreover, by improving Conrey and Ghosh's method, we can compute as many lower-order terms as desired.

This is joint work with Jonathan Bober (Bristol) and Micah Milinovich (Mississippi).

SAMPRIT GHOSH, University of Calgary
Certain Polytopes associated to Algebraic integer conjugates

In a recent paper, Bugeaud and Nguyen proved a stronger version of a Theorem of Lenstra and Shallit related to the convergents of certain algebraic integers. One of the key ingredients of their proof was certain exponential relations observed among the absolute values of the Galois conjugates of an algebraic integer. Motivated by their work, we let α be an algebraic integer of degree d and label its Galois conjugates $\alpha_0, \alpha_1, \cdots, \alpha_{d-1}$ written in decreasing order of magnitude, i.e. $|\alpha_0| \geq \cdots \geq |\alpha_{d-1}|$. Let $E_{k,d}$ be the set of $(c_1, \ldots, c_k) \in \mathbb{R}^k_{\geq 0}$ such that $|\alpha_0||\alpha_1|^{c_1}\cdots|\alpha_k|^{c_k} \geq 1$. In this talk we'll first give an explicit description of $E_{k,d}$ as a polytope with 2^k vertices. Then we will look at when the inequality is strict and give a quantitative version of the inequality depending on d and the height of the minimal polynomial of α. This is a joint work with S. Albayrak, G. Knapp and K.D. Nguyen.

NATHAN HEISZ, McMaster University
Densities of Bounded Primes in Hypergeometric Series

A Hypergeometric series ${}_mF_n(\alpha, \beta; z)$ is said to be p-adically bounded if the p-adic valuation of the coefficients is bounded below. A logical extension of this problem is to consider the Dirichelet density of bounded primes in a series with fixed parameters α and β. We will briefly summarize existing results from Franc et.al. on the densities of bounded primes for ${}_2F_1$ over $\mathbf{Q}$ before presenting new results on the densities of general ${}_mF_n$. Furthermore we will discuss a lower bound of the density of bounded primes in ${}_2F_1$ over quadratic number fields $\mathbf{Q}(\sqrt{D})$ and an interesting conjecture that gives an exact formulation for the densities in this case.

FATEMEZAHRA JANBAZI, University of Toronto
Boundedness of average rank of elliptic curves ordered by the coefficients

In arithmetic statistics, elliptic curves are typically ordered by the naive height, defined for $E_{A,B} : y^2 = x^3 + Ax + B$ as $H(E_{A,B}) = \max\{4|A|^3, 27B^2\}$, which effectively orders curves by the size of their roots. In this paper, we consider an alternative height function, $h(E_{A,B}) = \max\{|A|, |B|\}$, ordering elliptic curves by the magnitudes of their coefficients. We demonstrate that, under this new height function, the average size of the 2-Selmer group is bounded above by 3, aligning with the findings of Bhargava and Shankar under the naive height. We study the 2-Selmer group by analyzing integral binary quartic forms within non-uniformly expanding regions defined by the height function. Developing a new technique, we count and establish the equidistribution of lattice points in these spaces, overcoming challenges where standard methods fall short.

ABBAS MAAREFPARVAR, University of Lethbridge
An Application of Terada's Principal Ideal Theorem

For a number field K, denote by $\Gamma(K)$ the absolute genus field of K. In 2014, Amandine Leriche proved that if $K/\mathbb{Q}$ is an abelian extension, then the strongly ambiguous ideal class group of $\Gamma(K)/\mathbb{Q}$ is trivial. In this talk, we give a generalization of Leriche's result for finite cyclic extensions of number fields. More precisely, using Terada's Principal Ideal Theorem, we show that for a finite cyclic extension K/F, the strongly ambiguous ideal class group of $\Gamma(K/F)/F$ coincides with the image of the capitulation map from the ideal class group of F to the ideal class group of $\Gamma(K/F)$, where $\Gamma(K/F)$ denotes the relative genus field of K over F. This is a joint work with Ali Rajaei (Tarbiat Modares University) and Ehsan Shahoseini (Institute For Research In Fundamental Sciences).

BRETT NASSERDEN, McMaster University

PAUL PÉRINGUEY, University of British Columbia
Sign correlation between error terms of counting functions of primes in arithmetic progressions modulo 11

In this talk we will investigate the sign of the normalized error term for the primes in arithmetic progression, i.e the quantity $E^{\psi}(x;q,a) = \frac{\varphi(q)\psi(x;q,a)-x}{\sqrt{x}}$, where $\psi(x;q,a) = \sum_{\substack{n\leq x \\ n\equiv a \bmod q}} \Lambda(n)$ and Λ denotes the Von Mangoldt function.

More precisely, we study, under the Generalized Riemann Hypothesis and the Linear Independence Hypothesis, the logarithmic density of integers x for which $E^{\psi}(x;q,a)$ and $E^{\psi}(x;q,b)$ are of the same sign, for $(ab,q)=1$.

Furthermore we will provide numerical values for these densities when $q=11$.

This is a joint work with Kübra Benli and Greg Martin.

EMILY QUESADA-HERRERA, University of Lethbridge
Fourier optimization and quadratic forms

The study of integers and primes represented by binary quadratic forms is a classical problem, going back to Fermat. We will discuss a Fourier analysis approach to this problem, based on joint work with Andrés Chirre. For a given form and integer $\ell \geq 2$, this approach gives us strong estimates for the average number of representations of integers that are multiples of ℓ. This leads to unconditional upper bounds on the number of primes in short intervals represented by a given form, and, conditionally on the generalized Riemann hypothesis, an upper bound on the maximum gap between such consecutive primes. The latter extends a method of Carneiro, Milinovich, and Soundararajan.

JAXON SHUMAKER, University of Oregon
Classifying monogenic quartic orders

An order is a subring of the ring of integers of a number field. We say an order is *monogenic* if it is generated by a single element as an algebra over $\mathbb{Z}$. Following Bèrczes, Evertse, Győry, we consider two types of monogenic orders. A monogenic order is *type I* if there are two monogenizers ξ and β and there exist integers $a,b,c,d \in \mathbb{Z}$, with $c \neq 0$, such that $\beta = \dfrac{a\xi+b}{c\xi+d}$ and $ad-bc=\pm 1$. Additionally, a monogenic order is said to be type II if, for two generators ξ and β, there exist $f,g \in \mathbb{Z}[T]$, such that, $\deg(f) = \deg(g) = 2$ and $f(\xi) = \beta$, $g(\beta) = \xi$. It is proven by Bérczes, Evertse, and Győry that for a given number field, with certain conditions on the Galois group of the number field, nearly all the monogenic orders having at least two distinct generators, are type I or type II.

The work in this presentation is motivated by the following question "Do there exist monogenic quartic orders that are neither type I nor type II, and if so, can an explicit bound on the number of exceptional orders in a given quartic number field be given?" We use some of Akhtari's explicit results about index form equations in quartic number fields, to explore this question.

KIN MING TSANG, University of British Columbia
Comparing Hecke eigenvalues of automorphic representations for GL(2)

In this talk, we will discuss the strong multiplicity one theorem for $\mathrm{GL}(2)$, which basically states that if the local components of two cuspidal unitary automorphic representations are isomorphic for all but finitely many places, then they are globally equivalent. Ramakrishnan improved the result by showing that if two representations agree at places of Dirichlet density $7/8$, then they are globally equivalent. We will then discuss questions of similar flavour – comparing Hecke eigenvalues of two non-twist-equivalent cuspidal unitary automorphic representations.