Descriptive Set Theory, Continuous Logic, and Applications
(Org: **Christopher Eagle** (UVic), **Marcin Sabok** (McGill) and/et **Assaf Shani** (Concordia))

RACHAEL ALVIR, University of Waterloo
Scott Complexity and Torsion Abelian Groups

In this talk we review the connection between Scott sentences of countable structures and descriptive set theory. In particular, we show that the optimal Scott sentences of reduced Abelian p-Groups is arbitrarily high (below ω_1). In particular, this yields a new proof that the isomorphism relation on this class of structures is not a Borel equivalence relation. To do this, we first characterized the back-and-forth relations on this class of structures, which has many potential applications in computable structure theory. This work is joint with Luke MacLean and Barbara Csima.

JAN ARULSEELAN, McMaster University
Computability in Continuous Logic with Applications to Operator Algebras

We will discuss computable axiomatizations, computable presentations and their utility. We will sample some of the key theoretical hurdles to a working computable model theory of non-tracial von Neumann algebras and how they were overcome. We will then consider some directions for future work. Joint work with Isaac Goldbring, Bradd Hart, and Thomas Sinclair.

CHRISTOPHER KARPINSKI, McGill University
Hyperfiniteness of boundary actions of small cancellation groups

A metric space is *(Gromov) hyperbolic* if geodesic triangles in the metric space are uniformly slim. To any Gromov hyperbolic metric space, one can associate a boundary at infinity, called the *Gromov boundary*, which often has a natural Polish topology. A group acting on a hyperbolic metric space by isometries induces an action on the associated Gromov boundary by homeomorphisms. Given a hyperbolic space equipped with an action of a group, one can study the orbit equivalence relation of the boundary action. Interestingly, this orbit equivalence relation turns out to be hyperfinite in many cases (including for actions of free groups, and more generally hyperbolic groups, on the boundaries of their Cayley graphs). We show that a class of groups of interest in geometric group theory, the *small cancellation groups*, induce hyperfinite orbit equivalence relations on the boundaries of their natural hyperbolic Cayley graphs. This is joint work with Damian Osajda and Koichi Oyakawa.

JOSH LAU, University of Toronto
Algebro-topological invariants of co-existentially closed continua

We will discuss the Čech cohomology and pro-fundamental groups of compact Hausdorff spaces X for which the C*-algebra $C(X)$ of continuous functions on X is an existentially closed model of the theory of continua. This is joint work with C. Eagle and V. Marin-Marquez.

SAMUEL MURRAY, McGill
Borel Fractional Perfect Matchings in Quasitransitive Amenable Graphs

A fractional perfect matching is the linear programming analog of a perfect matching, where we allow edges to take on values in the interval $[0, 1]$ instead of just $\{0, 1\}$. Descriptive fractional perfect matchings have recently become an object of interest in descriptive combinatorics, as results of Bowen, Sabok, and Kun have shown that the existence of nice measurable fractional perfect matchings implies the existence of measurable perfect matchings in hyperfinite bipartite locally finite graphings.

A compactness argument shows that any locally finite hyperfinite graphing that admits a perfect matching will admit a measurable fractional perfect matching. However, in an upcoming paper by Bernshteyn and Weilacher, they construct a

polynomial growth Borel forest on a Polish space that has no Borel fractional perfect matching, even after throwing away an invariant meager set. In contrast to this result, we will show that if a Borel graph has components that are quasi-transitive and amenable, then if it admits a perfect matching it will admit a Borel fractional perfect matching.

TORI NOQUEZ, Saint Mary's College of California
Fractals as Final Coalgebras in Various Categories of Metric Spaces

In this talk we will explore a collection of results about obtaining fractal sets as final coalgebras of functors on various categories of metric spaces. This is a line of research started by Freyd and continued by Leinster, in which fractal sets are obtained as final coalgebras (a natural category theoretic construction used to capture infinite continuous behavior, such as streams) in categories of sets, and then topological spaces. The results presented here adapt this work to the metric setting, and examine this question in categories with different morphisms, namely short (non-expanding) maps, continuous maps, and Lipschitz maps. Interestingly, we obtain positive results for categories with short and continuous maps, but a negative result for the appropriate category of metric spaces with Lipschitz maps. We will conclude with a conjecture about a generalization of this negative result.

BO PENG, McGill University
Generalized Oxtoby systems and hyperfiniteness

We show that conjugacy relation of generalized Oxtoby systems is hyperfinite which generalizes Kaya's results to a kind of system where all Choquet Simplices can be realized as its invariant measures.

ANTOINE POULIN, McGill
Borel quasi-trees are treeable

In this talk, we consider countable Borel equivalence relations structured (in the sense of Chen-Kechris) by quasi-trees, that is graph quasi-isometric to trees. Using the Isbel-Werner duality between median graphs and (a class of) poset with complements, we show that all such equivalence relations are treeable.

FORTE SHINKO, University of California, Berkeley
Hyperfiniteness of graphs of slow intermediate growth

A definable graph on a standard Borel space is hyperfinite if it is the increasing union of component-finite definable subgraphs. Hyperfiniteness is a strong form of amenability, and it is a long-standing open problem to determine whether every amenable graph is in fact hyperfinite. We are quite far from resolving the problem, which is most notably still open for Schreier graphs of solvable groups, although there is a positive answer for nilpotent and polycyclic groups. Another natural class where the problem is open is graphs of subexponential growth, that is, graphs for which there is a subexponential function $f(n)$ such that every n-ball has at most $f(n)$ vertices. Recently, it was shown by Bernshteyn and Yu that every graph of polynomial growth is hyperfinite. We extend this to show that there is a constant $0 < c < 1$ such that every graph of growth $\exp(n^c)$ is hyperfinite. This is joint with Jan Grebík, Andrew Marks and Václav Rozhoň.

SPENCER UNGER, University of Toronto
Equidecomposition and discrepancy

We survey some recent results about equidecomposition which rely on discrepancy estimates for measures that come from actions of rotations on the torus. These results come from joint work with Andrew Marks and with Anton Bernshteyn and Anush Tserunyan.

ALLISON WANG, Carnegie Mellon University
Complexity of codes for Ramsey positive sets

A subset X of the Ellentuck space is called Ramsey null if given any non-empty basic open set $[s, A]$, there is some $B \in [s, A]$ such that $[s, B]$ and X are disjoint. A set is Ramsey positive if it is not Ramsey null. Sabok proved that in Ellentuck space, the set of codes for G_δ Ramsey positive sets is $\mathbf{\Sigma^1_2}$-complete. We build on Sabok's result to show that the same holds in the Milliken space of strong subtrees of the complete binary tree. In fact, we will see that the result holds for any topological Ramsey space satisfying a certain condition, including many common Ramsey spaces.

MICHAEL WOLMAN, Caltech
Invariant uniformization

Given sets X, Y and $P \subseteq X \times Y$ with $\mathrm{proj}_X(P) = X$, a uniformization of P is a function $f : X \to Y$ satisfying $(x, f(x)) \in P$ for $x \in X$. If E is an equivalence relation on X, say P is E-invariant if $x_1 E x_2 \implies P_{x_1} = P_{x_2}$, where $P_x = \{y : (x, y) \in P\}$ is the x-section of P. In this case, an E-invariant uniformization is a uniformization f satisfying $x_1 E x_2 \implies f(x_1) = f(x_2)$.

When X, Y are Polish spaces and P is Borel, standard results in descriptive set theory provide conditions which imply the existence of Borel uniformizations. These fall mainly into two categories: "small section" and "large section" results.

Suppose that E is a Borel equivalence relation on X, P is E-invariant, and P has "small" or "large" sections. We address the following question: When does there exist a Borel E-invariant uniformization of P?

We show that for a fixed E, every such P admits a Borel E-invariant uniformization iff E is smooth. Moreover, we compute the minimal definable complexity of counterexamples when E is not smooth. Our counterexamples use category, measure, and Ramsey-theoretic methods.

We also consider "local" dichotomies for such pairs (E, P). We give new proofs of a dichotomy of Miller in the case where P has countable sections, and prove anti-dichotomy results for the "large section" case. We discuss the "K_σ section" case, which is open.

This is joint with Alexander Kechris.

CLEMENT YUNG, University of Toronto
An alternative proof of the Mathias-Silver theorem using the Kastanas game

The Kastanas game was introduced by Kastanas as a game-theoretic characterisation of (completely) Ramsey subsets of $[\mathbb{N}]^\infty$. While, by Borel determinacy, this immediately implies the Galvin-Prikry theorem (every Borel subset of $[\mathbb{N}]^\infty$ is Ramsey), the characterisation alone is insufficient to conclude the Mathias-Silver theorem (every analytic subset of $[\mathbb{N}]^\infty$ is Ramsey). We prove that, by considering the same game in the space $[\mathbb{N}]^\infty \times 2^{\mathbb{N}}$, we may utilise this characterisation to conclude the Mathias-Silver theorem. We will also briefly discuss how we may apply this argument to weak A2 spaces, a class of spaces which includes topological Ramsey spaces and countable vector spaces.

ANDY ZUCKER, University of Waterloo
Topological groups with tractable minimal dynamics

In joint work with Gianluca Basso, we explore the class of Polish groups whose universal minimal flows admit a comeager orbit. By work of Ben Yaacov, Melleray, and Tsankov, this class contains all Polish groups with metrizable universal minimal flow, and by an example of Kwiatkowska, this inclusion is strict. We isolate the correct generalization of this class of Polish groups to the class of all topological groups. We call these the topological groups with "tractable minimal dynamics (TMD)." One way of phrasing what makes this class "tractable" is an "abstract Kechris-Pestov-Todorcevic correspondence," which characterizes membership in TMD using a Ramsey-theoretic property of the group. In particular, this implies that TMD is absolute between models of set theory. We also state some conjectures to the effect that any topological group not in TMD has "wild" minimal dynamics.