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Invariant uniformization

Given sets X, Y and $P \subseteq X \times Y$ with $\mathrm{proj}_X(P) = X$, a uniformization of P is a function $f : X \to Y$ satisfying $(x, f(x)) \in P$ for $x \in X$. If E is an equivalence relation on X, say P is E-invariant if $x_1 E x_2 \implies P_{x_1} = P_{x_2}$, where $P_x = \{y : (x, y) \in P\}$ is the x-section of P. In this case, an E-invariant uniformization is a uniformization f satisfying $x_1 E x_2 \implies f(x_1) = f(x_2)$.

When X, Y are Polish spaces and P is Borel, standard results in descriptive set theory provide conditions which imply the existence of Borel uniformizations. These fall mainly into two categories: "small section" and "large section" results.

Suppose that E is a Borel equivalence relation on X, P is E-invariant, and P has "small" or "large" sections. We address the following question: When does there exist a Borel E-invariant uniformization of P?

We show that for a fixed E, every such P admits a Borel E-invariant uniformization iff E is smooth. Moreover, we compute the minimal definable complexity of counterexamples when E is not smooth. Our counterexamples use category, measure, and Ramsey-theoretic methods.

We also consider "local" dichotomies for such pairs (E, P). We give new proofs of a dichotomy of Miller in the case where P has countable sections, and prove anti-dichotomy results for the "large section" case. We discuss the "K_σ section" case, which is open.

This is joint with Alexander Kechris.