Discrete Probability
(Org: **Sarai Hernandez-Torres** (Instituto de Matemáticas, UNAM) and/et **Gourab Ray** (University of Victoria))

JOHANNES BÄUMLER, UCLA
The truncation problem for long-range percolation

In long-range percolation on the integer lattice, for each pair of points {x,y}, there is an open edge between these points with probability depending on the Euclidean distance between the points, independent of all other edges. When are the long edges necessary for the existence of an infinite cluster? The truncation problem asks whether one can remove all long enough edges while still retaining an infinite open cluster. We discuss this question in the non-summable regime in dimensions $d \geq 3$. Here we show that the truncation problem has an affirmative answer.

HANNAH CAIRNS, McGill University
Cooperative motion in higher dimensions

Cooperative motion is a random walk process defined on a tree which has a recursive distributional equation. We discuss the scaling limit of the simple symmetric case of the process on the lattice $\mathbb{Z}^d$ for all dimensions $d \geq 1$. This is the first higher-dimensional result for this process. Joint work with Louigi Addario-Berry, Gavin Barill, and Jessica Lin.

KESAV KRISHNAN, University of Victoria
Local Convergence of Integer Valued Lipschitz Functions on Trees

The study of uniformly sampled integer valued Lipschitz functions of trees and related height function models has been of great recent interest. In particular, the phenomenon of localization, that is tightness of the law at the root has been established. In this talk, I will discuss joint work with Nathaniel Butler, Gourab Ray and Yinon Spinka that examines the local convergence of uniformly sampled 1-Lipschitz functions on $d-$ary trees which take the value zero on the leaves. In particular, as the number of generations goes to infinity, we show that local convergence holds if and only if $d < 8$. We also show that if the boundary values are allowed to be in $\{0,1\}$, then local convergence always holds via an FKG argument.

YUCHENG LIU, University of British Columbia
The torus plateau for the high-dimensional Ising model

We consider the Ising model on a d-dimensional discrete torus of volume r^d, in dimensions $d > 4$ and for large r, in the vicinity of the infinite-volume critical point β_c. We prove that for $\beta = \beta_c - \mathrm{const}\, r^{-d/2}$ (with a suitable constant) the susceptibility is bounded above and below by multiples of $r^{d/2}$, and that the two-point function has a "plateau" in the sense that it decays like $|x|^{-(d-2)}$ when $|x|$ is small relative to the volume but for larger $|x|$ it levels off to a constant value of order $r^{-d/2}$. We also prove that at $\beta = \beta_c - \mathrm{const}\, r^{-d/2}$ the renormalised coupling constant is nonzero, which implies a non-Gaussian limit for the average spin. The random current representation of the Ising model plays a central role in our analysis.

DANIEL DE LA RIVA MASSAAD, UBC
Voter Model stability with respect to conservative noises

The notions of noise sensitivity and stability were recently extended for the voter model, a well-known and studied interactive particle system. In this model, vertices of a graph have opinions that are updated by uniformly selecting edges. We further extend stability results to a different class of perturbations when an exclusion process or Brownian motions are performed in the collection of edge selections. We prove stability of the consensus opinion provided that the noise is being run for a short amount of time, which depends on the underlying graph structure. This is done by analyzing the expected size of the pivotal set, which needs to be properly defined for each setting.

MINGHAO PAN, Caltech
Dimension jump at the uniqueness threshold for percolation in $\infty + d$ dimensions

Consider percolation on $T \times \mathbb{Z}^d$, the product of a k-regular tree with the hypercubic lattice $\mathbb{Z}^d$. It is known that this graph has $p_c < p_u$, so that there are non-trivial regimes in which percolation has 0, ∞, and 1 infinite clusters a.s., and it was proven by Schonmann (1999) that there are infinitely many infinite clusters a.s. at the uniqueness threshold $p = p_u$. We strengthen this result significantly by showing that if $\delta_H(p)$ denotes the Hausdorff dimension of the set of accumulation points of an infinite cluster in the boundary of the tree then $\delta_H(p)$ has a jump discontinuity from $\delta_H(p_u) \leq 1/2$ to 1 at the uniqueness threshold p_u. We also prove that various other critical thresholds including the L^2 boundedness threshold $p_{2\to 2}$ coincide with p_u for such products, which are the first examples proven to have this property. All our results apply more generally to products of trees with arbitrary infinite amenable Cayley graphs and to the lamplighter on the tree.

LILY REEVES, California Institute of Technology
Phase Transitions of Ballistic Annihilation

Ballistic annihilation is a simple annihilating particle system motivated by the study of the kinetics of chemical reactions. In it, particles with presampled random velocities move across the real line and mutually annihilate upon collision. Many results were inferred by physicists, but it was only until recently that rigorous mathematical solutions were derived. In this talk, I will discuss Haslegrave—Sidoravicius—Tournier's breakthrough result in the symmetrical three-velocity setting and introduce two variants, for which we are able to prove the existence of phase transitions and compute the critical density despite considerably more complicated dynamics.