MINGHAO PAN, Caltech

Dimension jump at the uniqueness threshold for percolation in $\infty + d$ dimensions

Consider percolation on $T \times \mathbb{Z}^d$, the product of a k-regular tree with the hypercubic lattice $\mathbb{Z}^d$. It is known that this graph has $p_c < p_u$, so that there are non-trivial regimes in which percolation has 0, ∞, and 1 infinite clusters a.s., and it was proven by Schonmann (1999) that there are infinitely many infinite clusters a.s. at the uniqueness threshold $p = p_u$. We strengthen this result significantly by showing that if $\delta_H(p)$ denotes the Hausdorff dimension of the set of accumulation points of an infinite cluster in the boundary of the tree then $\delta_H(p)$ has a jump discontinuity from $\delta_H(p_u) \leq 1/2$ to 1 at the uniqueness threshold p_u. We also prove that various other critical thresholds including the L^2 boundedness threshold $p_{2\to 2}$ coincide with p_u for such products, which are the first examples proven to have this property. All our results apply more generally to products of trees with arbitrary infinite amenable Cayley graphs and to the lamplighter on the tree.