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Centrality of star factorizations

Let $\mathfrak{S}_n$ be the symmetric group on $\{1, 2, \ldots, n\}$ and $S := \{(i\, n) : 1 \leq i \leq n-1\}$. It is known that Cayley graph on $\mathfrak{S}_n$ with generating set S, denoted $\Gamma(\mathfrak{S}_n, S)$, has an interesting spectrum: all of its eigenvalues are integral. Walks on this Cayley graph beginning at the identity can be interpreted as *factorizations* of elements in $\mathfrak{S}_n$ where factors come from S. We call these *star factorizations*. Call a walk (or the corresponding factorization) *transitive* if the transpositions corresponding to the walk generate a transitive subgroup of $\mathfrak{S}_n$. The set of such walks in $\Gamma(\mathfrak{S}_n, S)$ have a remarkable property: the number of transitive walks to an element γ only depends on the conjugacy class of γ, a surprising result because of the asymmetry in the set S. We refer to this property | that the number of factorizations of conjugate elements is the same | as the *centrality* property. Goulden and Jackson (2009) showed that transitive star factorizations are central, but this result was obtained as a corollary of their enumerative formulae. They posed the natural problem of finding a combinatorial explanation for centrality. In this talk, I will discuss the centrality property of star factorizations and related factorizations that are central despite asymmetries in their underlying factorization problems. Our main result is that we give a combinatorial proof of the centrality of star factorizations, resolving the problem posed by Goulden and Jackson.

This is joint work with J. Campion Loth (Heilbronn Institute).