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Cover-free families on graphs

A family of subsets of $[t]$ is called a *d-cover-free family* (d-CFF) if no subset is contained in the union of any d others. We denote by $t(d, n)$ the minimum t for which there exists a d-CFF of $[t]$ with n subsets. $t(1, n)$ is determined using Sperner's Theorem. For $d \geq 2$, we rely on bounds for $t(d, n)$. Using the probabilistic approach, Erdös, Frankl, and Füredi proved $3.106 \log(n) < t(2, n) < 5.512 \log(n)$. Porat and Rothschild provided a deterministic polynomial-time algorithm to construct d-CFFs achieving $t = O(d^2 \log(n))$. Some upper bounds of $t(2, n)$(in some cases exact bounds) for some small values of n were provided by Li, van Rees, and Wei.

We extend the definition of 2-CFF to include a graph(G), called $\overline{G}$-CFF, where the edges of G specify the pair of subsets whose union must not cover any other subset. We denote by $t(G)$ as the minimum t for which there exists a $\overline{G}$-CFF. Thus, $t(K_n) = t(2, n)$. We will discuss some classical results on CFFs, along with constructions of $\overline{G}$-CFFs. We prove that for a graph G with n vertices, $t(1, n) \leq t(G) \leq t(2, n)$ and for an infinite family of star graphs with n vertices, $t(S_n) = t(1, n)$. We also provide constructions for $\overline{P_n}$-CFF and $\overline{C_n}$-CFF using a mixed-radix Gray code. This yields an upper bound for $t(P_n)$ and $t(C_n)$ that is smaller than the lower bound of $t(2, n)$ mentioned above.