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Symbolic computation of conservation laws of nonlinear partial differential equations

A direct method will be presented for the symbolic computation of conservation laws of nonlinear PDEs involving multiple space variables and time. Using the scaling symmetries of the PDE, the conserved densities are constructed as linear combinations of scaling homogeneous terms with undetermined coefficients. The variational derivative is used to compute the undetermined coefficients. The homotopy operator is used to invert the divergence operator, leading to an analytic expression of the flux vector.

The method is algorithmic and has been implemented in Mathematica. The software is being used to compute conservation laws of nonlinear PDEs occurring in the applied sciences and engineering. The software package will be demonstrated for PDEs that model, e.g., shallow water waves, ion-acoustic waves in plasmas, sound waves in nonlinear media, transonic gas flow, and stress and displacement in elastic materials. Examples include the Korteweg-de Vries and Zakharov-Kuznetsov equations, and a constitutive equation arising in elasticity.