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Tractable approximate solutions to a large family of 2nd order hyperbolic PDE

Traditional numerical methods, such as the method of lines, are often unsuitable for capturing the qualitative behavior of solutions to nonlinear PDEs, particularly due to high memory requirements and challenges with conservation properties and solution stability. These limitations become especially apparent in multi-dimensional systems or when dealing with solutions that develop discontinuities. Moreover, purely numerical solutions may diverge significantly when using finer spatial meshes or may fail to preserve the underlying conservation laws of the PDE system.

In this talk, we explore alternative approaches to deriving approximate yet analytically tractable PDEs from nonlinear perturbed systems of the form $u_{tt} - u_{xx} + \epsilon N(u) = 0$, where ϵ is a small parameter and N is nonlinear. Using methods such as multiple scales, we obtain reduced models that allow for a more manageable analysis of complex nonlinear phenomena, including shock formation and wave breaking. Numerical experiments, including error analysis and conservation testing, are presented for the dynamics governed by the PDE $u_{tt} = (1 + \epsilon u_x^2)u_{xx}$, where we can determine the break times, correct the multivalued solution, and track the shockwave. Similar tests are also performed after the addition of a nonlinear viscosity term: $u_{tt} = (1 + \epsilon u_x^2)u_{xx} + \eta(u_x^2 u_{xt})_x$