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Hidden symmetry groups in classical mechanics and beyond

Integrability and superintegrabitiy are important notions in n-dimensional classical Hamiltonian mechanics. A main example of superintegrable system in 3 dimensions comes from the Kepler potential, which possesses the Laplace-Runge-Lenz (LRL) vector as a global constant of motion in addition to energy and angular momentum. The Poisson bracket algebra of these constants of motion has the structure of the Lie algebra of the group $S0(4)$. This group represents a hidden symmetry structure since it is larger than the kinematic symmetry group $SO(3) \times R$ manifestly given by rotations and time translation.

This talk will explain how a similar hidden symmetry structure exists for all central force systems in n dimensions when local constants of motion are considered. In particular, every such system possesses $2n-1$ local constants of motion, including a generalized LRL vector, plus an integral of motion that depend explicitly on time. The latter quantity will be shown to lead to an enlarged symmetry group structure.A key tool is a version of Noether's theorem holding in the space of configuration variables extended to include time.

Mathematical and physical properties of the generalized LRL vector and the additional integral of motion, along with the associated symmetry group, will be presented.The n-dimensional Kepler potential and isotropic oscillator are used as examples.