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Exact Internal Waves in a Two-Fluid System

Although the Euler and Navier-Stokes fluid dynamics equations have been known for over 150 years, modern science is far from fully understanding their analytical properties. Exact and approximate solutions are available in a limited number of simplified cases, while direct numerical simulations are resource-intensive and often lack precision.

Many simplified models have been derived from Euler and Navier-Stokes equations to describe specific phenomena, such as surface and internal waves. These models aim to reduce the complexity of the original equations while preserving essential features of the phenomena and offering physical insight and computational accuracy. Examples include dimension and symmetry reductions, linearizations, and more general approximations based on asymptotic relationships. Fundamental nonlinear partial differential equations of mathematical physics, including Burgers', Korteweg-de Vries, nonlinear Schrödinger, and Kadomtsev-Petviashvili equations, arise in this context. Reduced models often reveal rich mathematical structures; their exact solutions can closely describe physical phenomena.

I will discuss a model of nonlinear internal waves in a stratified system of two non-mixing fluids with different densities, contained in a horizontal channel. This model, developed by Miyata and then Choi and Camassa, was derived through layer-averaging under the "shallow water" assumption, which assumes a small ratio of channel depth to wavelength, without requiring wave amplitudes to be small. I will introduce a transformation that simplifies the Choi-Camassa model, reducing it to a simpler dimensionless form, and demonstrate that the model admits simple physical exact solutions, including traveling waves, cnoidal waves, and kinks.