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Maximizing network connectivity subject to resource constraints

One way to measure how fast the network can propagate the information is using the Algebraic Connectivity (AC, or spectral gap) of a graph, which corresponds to the second eigenvalue of the Laplacian of the graph. We address the following question. Among all graphs of a given number of nodes n and edges m, which graph maximizes the AC? Generally, this question is very difficult for all but very small n and m (e.g. n=20, m=30).

For regular graphs, we derive attainable upper bounds on AC in terms of diameter and girth. Our diameter bound agrees with the well-known Alon-Boppana-Friedman bound for graphs of even diameter, but is an improvement for graphs of odd diameter. We then use a combination of stochastic algorithms and exhaustive search to find graphs which attain the diameter bound. For 3-regular graphs, we find attainable graphs for all diameters D up to and including D = 9 (the case of D = 10 is open). These graphs are extremely rare and also have high girth; for example we found exactly 45 distinct cubic graphs on 44 vertices attaining the upper bound when D = 7; all had girth 8 (out of a total of 266362 girth-8 graphs on 44 vertices).

We also derive an asymptotic bound for AC for several classes of random semi-regular graphs. In particular, we show that certain semi-regular graphs of average degree $d < 8$ are better than regular graphs of the same average degree, but regular graphs win when $d \geq 8$.